

```
> restart:
```

```
with(PDEtools, casesplit, declare):
with(DEtools, gensys):
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```
with(DifferentialGeometry):
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```
with(JetCalculus):
with(LieAlgebras):
with(GroupActions):
```

```
DGsetup([x,y,z,u], Rquatre):      Repere_xyzu := evalDG([D_x,
D_y,D_z,D_u]);
```

```
XX := (-(5/2)*F050*T2-(1/2)*T3)*x+T1;
```

```
YY := (-T3-(5/2)*F050*T2)*y-T2*z+T2;
```

```
ZZ := -4*T2*y-T3*z+T3;
```

```
UU := 2*T1*x+2*T2*y+(-5*F050*T2-T3)*u;
```

```
FF := sort(expand(
```

```
x^2+y^2+y^2*z+y^4+y^2*z^2+F050*y^5+3*y^4*z+y^2*z^3+((5/4)*
F050^2+2)*y^6+4*F050*y^5*z+6*y^4*z^2+y^2*z^4+((25/14)*F050^3+
(36/7)*F050)*y^7+((25/4)*F050^2+10)*y^6*z+10*F050*y^5*z^2+10*y^4*
z^3+y^2*z^5
```

```
), [z,y,x], ascending);
```

```
indets({XX,YY,ZZ,UU}) minus {T1,T2,T3,x,y,z,u};
```

$$\text{Repere\_xyzu} := [\partial_x, \partial_y, \partial_z, \partial_u]$$

$$XX := \left( -\frac{5}{2} F050 T2 - \frac{1}{2} T3 \right) x + T1$$

$$YY := \left( -T3 - \frac{5}{2} F050 T2 \right) y - T2 z + T2$$

$$ZZ := -4 T2 y - T3 z + T3$$

$$UU := 2 T1 x + 2 T2 y + (-5 F050 T2 - T3) u$$

$$\begin{aligned}
 FF := & x^2 + y^2 + z y^2 + y^4 + z^2 y^2 + F050 y^5 + 3 z y^4 + z^3 y^2 + \frac{5}{4} F050^2 y^6 + 2 y^6 \\
 & + 4 F050 z y^5 + 6 z^2 y^4 + z^4 y^2 + \frac{25}{14} F050^3 y^7 + \frac{36}{7} F050 y^7 + \frac{25}{4} F050^2 z y^6 \\
 & + 10 z y^6 + 10 F050 z^2 y^5 + 10 z^3 y^4 + z^5 y^2 \\
 & \{F050\}
 \end{aligned} \tag{1}$$

$$\begin{aligned}
 & \text{> LL := evalDG(} \\
 & \quad \text{XX*D_x} \\
 & \quad + \\
 & \quad \text{YY*D_y} \\
 & \quad + \\
 & \quad \text{ZZ*D_z} \\
 & \quad + \\
 & \quad \text{UU*D_u} \\
 & \quad \text{);} \\
 & \quad \text{Composantes_LL := GetComponent( LL, Repere_xyzu):} \\
 & \quad \text{LL := } - \left( \frac{5}{2} x F050 T2 + \frac{1}{2} x T3 - T1 \right) \partial_x - \left( y T3 + \frac{5}{2} y F050 T2 + T2 z - T2 \right) \partial_y - \\
 & \quad \left( 4 T2 y + T3 z - T3 \right) \partial_z - \left( 5 F050 T2 u - 2 T1 x - 2 T2 y + T3 u \right) \partial_u
 \end{aligned} \tag{2}$$

$$\begin{aligned}
 & \text{> e[0] := evalDG(subs({T1=0,T2=0,T3=0}, LL));} \\
 & \quad e_0 := 0 \partial_x + 0 \partial_y + 0 \partial_z + 0 \partial_u
 \end{aligned} \tag{3}$$

$$\text{> e[1] := evalDG(subs({T1=1,T2=0,T3=0}, LL));}$$

```
e[2] := evalDG(subs({T1=0,T2=1,T3=0}, LL));
e[3] := evalDG(subs({T1=0,T2=0,T3=1}, LL));
```

$$\begin{aligned}
 e_1 &:= \partial_x + 0 \partial_y + 0 \partial_z + 2x \partial_u \\
 e_2 &:= -\frac{5x F050}{2} \partial_x - \left(-1 + \frac{5y F050}{2} + z\right) \partial_y - 4y \partial_z - \left(5 F050 u - 2y\right) \partial_u \\
 e_3 &:= -\frac{x}{2} \partial_x - y \partial_y - (z-1) \partial_z - u \partial_u
 \end{aligned}$$

(4)

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> algebre_lie := LieAlgebraData([seq(e[i], i=1..3)]);
```

```
DGsetup(algebre_lie):
```

```
LD := LeviDecomposition();
```

```
resoluble := Query("Solvable");
```

```
semi_simple := Query("Semisimple");
```

```
MultiplicationTable("LieTable");
```

$$algebre\_lie := \left[ e1, e2 \right] = -\frac{5 F050}{2} e1, \left[ e1, e3 \right] = -\frac{1}{2} e1, [e2, e3] = 0$$

$$LD := [[e1, e2, e3], [ ]]$$

$$resoluble := true$$

$$semi\_simple := false$$

L1	$e1$	$e2$	$e3$
$e1$	0	$-\frac{5 F050}{2} e1$	$-\frac{1}{2} e1$
$e2$	$\frac{5 F050}{2} e1$	0	0
$e3$	$\frac{1}{2} e1$	0	0

(5)

