

```
> restart:
```

```
with(PDEtools, casesplit, declare):
with(DEtools, gensys):
```

```
with(DifferentialGeometry):
```

```
with(JetCalculus):
with(LieAlgebras):
with(GroupActions):
```

```
DGsetup([x,y,z,u], Rquatre):      Repere_xyzu := evalDG([D_x,
D_y,D_z,D_u]);
```

```
FF := x^2+y^2+(1/2)*x^4+x^2*y^2+(1/2)*y^4+(1/2)*x^6+(3/2)*x^4*
y^2+(3/2)*x^2*y^4+(1/2)*y^6;
```

$Repere_xyzu := [\partial_x, \partial_y, \partial_z, \partial_u]$

$$FF := x^2 + y^2 + \frac{1}{2} x^4 + x^2 y^2 + \frac{1}{2} y^4 + \frac{1}{2} x^6 + \frac{3}{2} x^4 y^2 + \frac{3}{2} x^2 y^4 + \frac{1}{2} y^6 \quad (1)$$

```
> e[1] := evalDG(-(u-1)*D_x+0*D_y+0*D_z+2*x*D_u);
e[2] := evalDG(0*D_x-(u-1)*D_y+0*D_z+2*y*D_u);
e[3] := evalDG(0*D_x+0*D_y+D_z+0*D_u);
```

```
e[4] := evalDG(y*D_x-x*D_y+0*D_z+0*D_u);
e[5] := evalDG(0*D_x+0*D_y+x*D_z+0*D_u);
e[6] := evalDG(0*D_x+0*D_y+y*D_z+0*D_u);
e[7] := evalDG(0*D_x+0*D_y+z*D_z+0*D_u);
e[8] := evalDG(0*D_x+0*D_y+u*D_z+0*D_u);
```

$$e_1 := -(u-1) \partial_x + 2x \partial_u$$

$$e_2 := -(u-1) \partial_y + 2y \partial_u$$

$$e_3 := \partial_z$$

$$e_4 := y \partial_x - x \partial_y$$

$$e_5 := x \partial_z$$

$$e_6 := y \partial_z$$

$$e_7 := z \partial_z$$

$$e_8 := u \partial_z$$

(2)

```
> algebre_lie := LieAlgebraData([seq(e[i], i=1..8)]);
```

```
  DGsetup(algebre_lie):
```

```
  LD := LeviDecomposition();
```

```
  resolvable := Query("Solvable");
```

```
  semi_simple := Query("Semisimple");
```

```
  MultiplicationTable("LieTable");
```

algebre_lie := $[e1, e2] = 2 e4, [e1, e3] = 0, [e1, e4] = -e2, [e1, e5] = e3 - e8,$
 $[e1, e6] = 0, [e1, e7] = 0, [e1, e8] = 2 e5, [e2, e3] = 0, [e2, e4] = e1, [e2, e5]$
 $] = 0, [e2, e6] = e3 - e8, [e2, e7] = 0, [e2, e8] = 2 e6, [e3, e4] = 0, [e3, e5]$
 $] = 0, [e3, e6] = 0, [e3, e7] = e3, [e3, e8] = 0, [e4, e5] = e6, [e4, e6] = -e5,$
 $[e4, e7] = 0, [e4, e8] = 0, [e5, e6] = 0, [e5, e7] = e5, [e5, e8] = 0, [e6, e7]$
 $] = e6, [e6, e8] = 0, [e7, e8] = -e8$

LD := $[[e3, e5, e6, e7, e8], [e1, e2, e4]]$

resolvable := false

semi_simple := false

| L1 | <i>e1</i> | <i>e2</i> | <i>e3</i> | <i>e4</i> | <i>e5</i> | <i>e6</i> | <i>e7</i> | <i>e8</i> |
|-----------|-------------------------|-------------------------|-------------|-------------|-----------------------|-----------------------|-----------|-------------|
| <i>e1</i> | 0 | 2 <i>e4</i> | 0 | - <i>e2</i> | <i>e3</i> - <i>e8</i> | 0 | 0 | 2 <i>e5</i> |
| <i>e2</i> | -2 <i>e4</i> | 0 | 0 | <i>e1</i> | 0 | <i>e3</i> - <i>e8</i> | 0 | 2 <i>e6</i> |
| <i>e3</i> | 0 | 0 | 0 | 0 | 0 | 0 | <i>e3</i> | 0 |
| <i>e4</i> | <i>e2</i> | - <i>e1</i> | 0 | 0 | <i>e6</i> | - <i>e5</i> | 0 | 0 |
| <i>e5</i> | - <i>e3</i> + <i>e8</i> | 0 | 0 | - <i>e6</i> | 0 | 0 | <i>e5</i> | 0 |
| <i>e6</i> | 0 | - <i>e3</i> + <i>e8</i> | 0 | <i>e5</i> | 0 | 0 | <i>e6</i> | 0 |
| <i>e7</i> | 0 | 0 | - <i>e3</i> | 0 | - <i>e5</i> | - <i>e6</i> | 0 | - <i>e8</i> |
| <i>e8</i> | -2 <i>e5</i> | -2 <i>e6</i> | 0 | 0 | 0 | 0 | <i>e8</i> | 0 |

(3)

