

```

> restart:

with(PDEtools, casesplit, declare):
with(DEtools, gensys):

with(DifferentialGeometry):

with(JetCalculus):
with(LieAlgebras):
with(GroupActions):

DGsetup([x,y,z,u], Rquatre):      Repere_xyzu := evalDG([D_x,
D_y,D_z,D_u]);

FF := x^2+y^2;

```

$$\begin{aligned}
 \text{Repere_xyzu} &:= [\partial_x, \partial_y, \partial_z, \partial_u] \\
 FF &:= x^2 + y^2
 \end{aligned}$$

(1)

```

> e[1] := evalDG(D_x+0*D_y+0*D_z+2*x*D_u);
e[2] := evalDG(0*D_x+D_y+0*D_z+2*y*D_u);
e[3] := evalDG(0*D_x+0*D_y+D_z+0*D_u);

e[4] := evalDG(x*D_x+y*D_y+0*D_z+2*u*D_u);
e[5] := evalDG(y*D_x-x*D_y+0*D_z+0*D_u);
e[6] := evalDG(0*D_x+0*D_y+x*D_z+0*D_u);
e[7] := evalDG(0*D_x+0*D_y+y*D_z+0*D_u);
e[8] := evalDG(0*D_x+0*D_y+z*D_z+0*D_u);
e[9] := evalDG(0*D_x+0*D_y+u*D_z+0*D_u);


$$\begin{aligned}
 e_1 &:= \partial_x + 2x\partial_u \\
 e_2 &:= \partial_y + 2y\partial_u \\
 e_3 &:= \partial_z \\
 e_4 &:= x\partial_x + y\partial_y + 2u\partial_u \\
 e_5 &:= y\partial_x - x\partial_y \\
 e_6 &:= x\partial_z
 \end{aligned}$$


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$$e_7 := y \partial_z$$

$$e_8 := z \partial_z$$

$$e_9 := u \partial_z$$

(2)

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> algebre_lie := LieAlgebraData([seq(e[i], i=1..9)]);
```

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  DGsetup(algebre_lie):
```

```
  LD := LeviDecomposition();
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```
  resolvable := Query("Solvable");
```

```
  semi_simple := Query("Semisimple");
```

```
  MultiplicationTable("LieTable");
```

```
algebre_lie := [e1, e2] = 0, [e1, e3] = 0, [e1, e4] = e1, [e1, e5] = -e2, [e1, e6]
               ] = e3, [e1, e7] = 0, [e1, e8] = 0, [e1, e9] = 2 e6, [e2, e3] = 0, [e2, e4] = e2,
               [e2, e5] = e1, [e2, e6] = 0, [e2, e7] = e3, [e2, e8] = 0, [e2, e9] = 2 e7, [e3, e4]
               ] = 0, [e3, e5] = 0, [e3, e6] = 0, [e3, e7] = 0, [e3, e8] = e3, [e3, e9] = 0, [e4,
               e5] = 0, [e4, e6] = e6, [e4, e7] = e7, [e4, e8] = 0, [e4, e9] = 2 e9, [e5, e6]
               ] = e7, [e5, e7] = -e6, [e5, e8] = 0, [e5, e9] = 0, [e6, e7] = 0, [e6, e8] = e6,
               [e6, e9] = 0, [e7, e8] = e7, [e7, e9] = 0, [e8, e9] = -e9
```

```
      LD := [[e1, e2, e3, e4, e5, e6, e7, e8, e9], [ ]]
```

```
      resolvable := true
```

```
      semi_simple := false
```

(3)

L1	$e1$	$e2$	$e3$	$e4$	$e5$	$e6$	$e7$	$e8$	$e9$
$e1$	0	0	0	$e1$	$-e2$	$e3$	0	0	$2e6$
$e2$	0	0	0	$e2$	$e1$	0	$e3$	0	$2e7$
$e3$	0	0	0	0	0	0	0	$e3$	0
$e4$	$-e1$	$-e2$	0	0	0	$e6$	$e7$	0	$2e9$
$e5$	$e2$	$-e1$	0	0	0	$e7$	$-e6$	0	0
$e6$	$-e3$	0	0	$-e6$	$-e7$	0	0	$e6$	0
$e7$	0	$-e3$	0	$-e7$	$e6$	0	0	$e7$	0
$e8$	0	0	$-e3$	0	0	$-e6$	$-e7$	0	$-e9$
$e9$	$-2e6$	$-2e7$	0	$-2e9$	0	0	0	$e9$	0

(3)