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> restart:
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```
with(PDEtools, casesplit, declare):
with(DEtools, gensys):
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with(DifferentialGeometry):
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with(JetCalculus):
with(LieAlgebras):
with(GroupActions):
```

```
DGsetup([x,y,z,u], Rquatre):      Repere_xyzu := evalDG([D_x,
D_y,D_z,D_u]);
```

```
FF := sort(expand(
```

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2*x^3*z+3*x^3*z^2+4*x^3*z^3+x^2*z+y^2*z+F400*x^4+F130*x*y^3+x^2*
z^2+y^2*z^2+(-(13/90)*F130^2+(1/5)*F320+(8/5)*F400^2-(3/5)*F400)*
x^5+((5/3)*F050+(9/4)*F130-2*F130*F400)*x^4*y+F320*x^3*y^2+(4*
F130*F400-(15/4)*F130)*x^2*y^3+F050*y^5+3*F130*x*y^3*z+x^2*z^3+
y^2*z^3+(-(11/120)*F130^2+(3/10)*F320+(3/5)*F400+(5/54)*F050*F130
-(14/5)*F400^2+(16/5)*F400^3-(2/5)*F400*F130^2+(2/5)*F400*F320)*
x^6+(-(27/8)*F130+12*F130*F400+(1/15)*F130*F320+(13/270)*F130^3-
(5/2)*F050+(20/3)*F050*F400-8*F130*F400^2)*x^5*y+(-(83/16)*F130^2
-(9/4)*F320-(55/18)*F050*F130+(83/18)*F400*F130^2+4*F400*F320)*
x^4*y^2+((105/8)*F130-29*F130*F400-(4/3)*F130*F320-(10/9)*
F130^3+16*F130*F400^2)*x^3*y^3+(-12*F400*F130^2+(5/2)*F050*F130+
(27/2)*F130^2)*x^2*y^4+((17/18)*F130^3+F130*F320-(27/2)*F050+12*
F050*F400)*x*y^5+(-(37/18)*F050*F130+(2/3)*F400*F130^2-(5/8)*
F130^2)*y^6+(-(26/45)*F130^2+(4/5)*F320-(12/5)*F400+(32/5)*
F400^2)*x^5*z+((20/3)*F050+9*F130-8*F130*F400)*x^4*y*z+4*F320*
x^3*y^2*z+(16*F130*F400-15*F130)*x^2*y^3*z-6*F130^2*x*y^4*z+6*
F130*x*y^3*z^2+x^2*z^4+y^2*z^4-(3/2)*F130^2*x*y^4+3*F400*x^4*z+4*
F050*y^5*z+6*F400*x^4*z^2+x^2+y^2+x^3
```

```
), [z,y,x], ascending);
```

$$Repere_xyzu := [\partial_x, \partial_y, \partial_z, \partial_u]$$

$$FF := x^2 + y^2 + x^3 + z x^2 + z y^2 + F400 x^4 + F130 y^3 x + 2 z x^3 + z^2 x^2 + z^2 y^2 - \frac{13}{90} F130^2 x^5 + \frac{1}{5} F320 x^5 + \frac{8}{5} F400^2 x^5 - \frac{3}{5} F400 x^5 + \frac{5}{3} F050 y x^4 \quad (1)$$

$$\begin{aligned}
& + \frac{9}{4} F130 y x^4 - 2 F130 F400 y x^4 + F320 y^2 x^3 - \frac{15}{4} F130 y^3 x^2 \\
& + 4 F130 F400 y^3 x^2 - \frac{3}{2} F130^2 y^4 x + F050 y^5 + 3 F400 z x^4 + 3 F130 z y^3 x \\
& + 3 z^2 x^3 + z^3 x^2 + z^3 y^2 + \frac{5}{54} F050 F130 x^6 - \frac{2}{5} F400 F130^2 x^6 \\
& + \frac{2}{5} F400 F320 x^6 - \frac{11}{120} F130^2 x^6 + \frac{3}{10} F320 x^6 + \frac{3}{5} F400 x^6 - \frac{14}{5} F400^2 x^6 \\
& + \frac{16}{5} F400^3 x^6 - \frac{27}{8} F130 y x^5 + \frac{13}{270} F130^3 y x^5 - \frac{5}{2} F050 y x^5 \\
& + 12 F130 F400 y x^5 + \frac{1}{15} F130 F320 y x^5 + \frac{20}{3} F050 F400 y x^5 \\
& - 8 F130 F400^2 y x^5 - \frac{83}{16} F130^2 y^2 x^4 - \frac{9}{4} F320 y^2 x^4 - \frac{55}{18} F050 F130 y^2 x^4 \\
& + \frac{83}{18} F400 F130^2 y^2 x^4 + 4 F400 F320 y^2 x^4 + \frac{105}{8} F130 y^3 x^3 \\
& - \frac{10}{9} F130^3 y^3 x^3 - 29 F130 F400 y^3 x^3 - \frac{4}{3} F130 F320 y^3 x^3 \\
& + 16 F130 F400^2 y^3 x^3 + \frac{27}{2} F130^2 y^4 x^2 - 12 F400 F130^2 y^4 x^2 \\
& + \frac{5}{2} F050 F130 y^4 x^2 + \frac{17}{18} F130^3 y^5 x - \frac{27}{2} F050 y^5 x + F130 F320 y^5 x \\
& + 12 F050 F400 y^5 x - \frac{37}{18} F050 F130 y^6 + \frac{2}{3} F400 F130^2 y^6 - \frac{5}{8} F130^2 y^6 \\
& - \frac{26}{45} F130^2 z x^5 + \frac{4}{5} F320 z x^5 - \frac{12}{5} F400 z x^5 + \frac{32}{5} F400^2 z x^5 \\
& - 8 F130 F400 z y x^4 + \frac{20}{3} F050 z y x^4 + 9 F130 z y x^4 + 4 F320 z y^2 x^3 \\
& + 16 F130 F400 z y^3 x^2 - 15 F130 z y^3 x^2 - 6 F130^2 z y^4 x + 4 F050 z y^5 \\
& + 6 F400 z^2 x^4 + 6 F130 z^2 y^3 x + 4 z^3 x^3 + z^4 x^2 + z^4 y^2
\end{aligned}$$

```

> e1 := evalDG(-(-1+4*F400*x-3*x-(1/3)*F130*y+z+(4/9)*u*F130^2+u*
F320)*D_x+(-(1/3)*F130*x+(9/2)*y-4*y*F400+(7/2)*u*F130-(10/3)*u*
F130*F400+(10/3)*u*F050)*D_y-(-(8/9)*x*F130^2-2*x*F320-(20/3)*y*
F130*F400+(20/3)*y*F050+8*F130*y-(7/6)*u*F130^2)*D_z-(8*F400*u-9*
u-2*x)*D_u);

```

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e2 := evalDG((3*F130*x+(10/3)*u*F050-4*u*F130*F400+(15/4)*u*F130)
*D_x+(3*F130*y+F320*u-z+1)*D_y-((20/3)*x*F050+(21/2)*F130*x-8*x*

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F130*F400+2*y*F320+5*u*F050)*D_z+(6*F130*u+2*y)*D_u);

e3 := evalDG(-x*D_x-y*D_y-(-1+z)*D_z-u*D_u);

$$\begin{aligned}
 e1 := & -\left(-1 + 4 F400 x - 3 x - \frac{1}{3} F130 y + z + \frac{4}{9} u F130^2 + u F320\right) \partial_x + \left(-\frac{1}{3} F130 x + \frac{9}{2} y - 4 y F400 + \frac{7}{2} u F130 - \frac{10}{3} u F130 F400 + \frac{10}{3} u F050 \right) \partial_y \\
 & - \left(-\frac{8}{9} x F130^2 - 2 x F320 - \frac{20}{3} y F130 F400 + \frac{20}{3} y F050 + 8 F130 y - \frac{7}{6} u F130^2 \right) \partial_z - \left(8 F400 u - 9 u - 2 x \right) \partial_u \\
 e2 := & \left(3 F130 x + \frac{10}{3} u F050 - 4 u F130 F400 + \frac{15}{4} u F130 \right) \partial_x + \left(3 F130 y + u F320 - z + 1 \right) \partial_y - \left(\frac{20}{3} x F050 + \frac{21}{2} F130 x - 8 x F130 F400 + 2 y F320 + 5 u F050 \right) \partial_z + \left(6 u F130 + 2 y \right) \partial_u \\
 e3 := & -x \partial_x - y \partial_y - (-1 + z) \partial_z - u \partial_u
 \end{aligned}
 \tag{2}$$

> ## 5 GENERATEURS

B[1] := factor(60672*F130*F400^2-17920*F050*F400+12288*F130*F320-136512*F130*F400+20160*F050+74655*F130);

B[2] := factor(632*F130^2*F400-640*F050*F130-711*F130^2-288*F320*F400+324*F320);

B[3] := factor(43520*F050*F130*F400+12288*F130^2*F320+27648*F320*F400^2-48960*F050*F130-2133*F130^2-62208*F320*F400+34992*F320);

B[4] := factor(F050*(43520*F050*F400+16384*F130*F320-48960*F050-2133*F130));

B[5] := factor(158*F130^3-1080*F050*F400-72*F130*F320+1215*F050);

$$\begin{aligned}
 B_1 := & 60672 F130 F400^2 - 17920 F050 F400 + 12288 F130 F320 \\
 & - 136512 F130 F400 + 20160 F050 + 74655 F130
 \end{aligned}$$

$$B_2 := 632 F130^2 F400 - 640 F050 F130 - 711 F130^2 - 288 F320 F400 + 324 F320$$

$$\begin{aligned}
B_3 &:= 43520 F050 F130 F400 + 12288 F130^2 F320 + 27648 F320 F400^2 \\
&\quad - 48960 F050 F130 - 2133 F130^2 - 62208 F320 F400 + 34992 F320 \\
B_4 &:= F050 (43520 F050 F400 + 16384 F130 F320 - 48960 F050 - 2133 F130) \\
B_5 &:= 158 F130^3 - 1080 F050 F400 - 72 F130 F320 + 1215 F050
\end{aligned} \tag{3}$$

> ## BASE DE GROEBNER COMPLETE

with(Groebner):

Ordre := tdeg:

Variables := {F050, F130, F320, F400}:

VariablesOrdonnees := Ordre(seq(op(j,Variables),j=1..nops(Variables))):

Ideal := {seq(B[i], i=1..5)}:

Hdim := HilbertDimension(Ideal);

Base := Basis(Ideal, VariablesOrdonnees):

NbBase := nops(Base);

for i from 1 to nops(Base) do

 rB[i] := factor(op(i,Base))

od;

Ensemble_Base := {seq(op(i,Base), i=1..nops(Base))}:

Hdim:= 2

NbBase:= 10

$$rB_1 := 60672 F130 F400^2 - 17920 F050 F400 + 12288 F130 F320$$

$$- 136512 F130 F400 + 20160 F050 + 74655 F130$$

$$rB_2 := 632 F130^2 F400 - 640 F050 F130 - 711 F130^2 - 288 F320 F400 + 324 F320$$

$$rB_3 := 43520 F050 F130 F400 + 12288 F130^2 F320 + 27648 F320 F400^2$$

$$- 48960 F050 F130 - 2133 F130^2 - 62208 F320 F400 + 34992 F320$$

$$rB_4 := F050 (43520 F050 F400 + 16384 F130 F320 - 48960 F050 - 2133 F130)$$

$$rB_5 := 158 F130^3 - 1080 F050 F400 - 72 F130 F320 + 1215 F050$$

$$rB_6 := F050 (256 F130^2 - 1728 F400^2 + 3888 F400 - 2187)$$

$$\begin{aligned}
rB_7 &:= F050 (1392640 F050 F130 + 3538944 F320 F400 - 3981312 F320 \\
&\quad - 460728 F400 + 518319) \\
rB_8 &:= 13105152 F320 F400^3 + 1523200 F050^2 F400 + 2654208 F320^2 F400 \\
&\quad - 44229888 F320 F400^2 - 1713600 F050^2 - 74655 F050 F130 \\
&\quad - 2985984 F320^2 + 49297896 F320 F400 - 18141165 F320 \\
rB_9 &:= F050 (8 F400 - 9) (3264 F400^2 + 1024 F320 - 7344 F400 + 3969) \\
rB_{10} &:= F050 (7575961600 F050^2 - 7247757312 F320^2 + 1887141888 F320 \\
&\quad - 122841603) (8 F400 - 9)
\end{aligned} \tag{4}$$

$$\begin{aligned}
&> \text{Tableau_crochets} := \text{Matrix}(3, 3, \{(1, 1) = 0, (1, 2) = (8/3)* \\
&\quad F130*ee1+(-9/2+4*F400)*ee2+((4/3)*F130*F400-(5/2)*F130)*ee3, (1, \\
&\quad 3) = 0, (2, 1) = -(8/3)*F130*ee1-(-9/2+4*F400)*ee2-((4/3)*F130* \\
&\quad F400-(5/2)*F130)*ee3, (2, 2) = 0, (2, 3) = 0, (3, 1) = 0, (3, 2) \\
&\quad = 0, (3, 3) = 0\}); \\
\text{Tableau_crochets} &:= \left[\left[0, \frac{8}{3} F130 ee1 + \left(-\frac{9}{2} + 4 F400 \right) ee2 + \left(\frac{4}{3} F130 F400 \right. \right. \right. \\
&\quad \left. \left. - \frac{5}{2} F130 \right) ee3, 0 \right], \right. \\
&\quad \left[-\frac{8}{3} F130 ee1 - \left(-\frac{9}{2} + 4 F400 \right) ee2 - \left(\frac{4}{3} F130 F400 - \frac{5}{2} F130 \right) ee3, 0, 0 \right], \\
&\quad \left[0, 0, 0 \right] \right]
\end{aligned} \tag{5}$$