

```
> restart:
```

```
with(PDEtools, casesplit, declare):
with(DEtools, gensys):
```

```
with(DifferentialGeometry):
```

```
with(JetCalculus):
with(LieAlgebras):
with(GroupActions):
```

```
DGsetup([x,y,z,u], Rquatre):      Repere_xyzu := evalDG([D_x,
D_y,D_z,D_u]);
```

```
XX := (-(3/2)*T3-3*T2)*x+T1;
```

```
YY := (-2*T3-3*T2)*y-T2*z+T2;
```

```
ZZ := (-T2-T3)*y+(-3*T2-T3)*z+T3;
```

```
UU := 2*T1*x+2*T2*y+(-3*T3-6*T2)*u;
```

```
FF := sort(expand(
```

```
x^2+y^2+y^2*z+(1/4)*y^4+y^3*z+y^2*z^2+(1/2)*y^5+(9/4)*y^4*z+3*
y^3*z^2+y^2*z^3+(9/8)*y^6+(23/4)*y^5*z+(39/4)*y^4*z^2+6*y^3*z^3+
y^2*z^4+(11/4)*y^7+(127/8)*y^6*z+(131/4)*y^5*z^2+(115/4)*y^4*
z^3+10*y^3*z^4+y^2*z^5
```

```
), [z,y,x], ascending);
```

```
indets({XX,YY,ZZ,UU}) minus {T1,T2,T3,x,y,z,u};
```

$$\text{Repere_xyzu} := [\partial_x, \partial_y, \partial_z, \partial_u]$$

$$XX := \left(-\frac{3}{2} T3 - 3 T2 \right) x + T1$$

$$YY := (-2 T3 - 3 T2) y - T2 z + T2$$

$$\begin{aligned}
ZZ &:= (-T2 - T3) y + (-3 T2 - T3) z + T3 \\
UU &:= 2 T1 x + 2 T2 y + (-3 T3 - 6 T2) u \\
FF &:= x^2 + y^2 + z y^2 + \frac{1}{4} y^4 + z y^3 + z^2 y^2 + \frac{1}{2} y^5 + \frac{9}{4} z y^4 + 3 z^2 y^3 + z^3 y^2 + \frac{9}{8} y^6 \\
&\quad + \frac{23}{4} z y^5 + \frac{39}{4} z^2 y^4 + 6 z^3 y^3 + z^4 y^2 + \frac{11}{4} y^7 + \frac{127}{8} z y^6 + \frac{131}{4} z^2 y^5 \\
&\quad + \frac{115}{4} z^3 y^4 + 10 z^4 y^3 + z^5 y^2
\end{aligned}
\tag{1}$$

$$\begin{aligned}
&> LL := \text{evalDG}(\\
&\quad XX * D_x \\
&\quad + \\
&\quad YY * D_y \\
&\quad + \\
&\quad ZZ * D_z \\
&\quad + \\
&\quad UU * D_u \\
&\quad) ; \\
&\quad \text{Composantes_LL} := \text{GetComponents}(LL, \text{Repere_xyz}u); \\
LL &:= \left(-\frac{3}{2} x T3 - 3 x T2 + T1 \right) \partial_x - \left(3 T2 y + T2 z + 2 y T3 - T2 \right) \partial_y - \left(T2 y + 3 T2 z \right. \\
&\quad \left. + y T3 + z T3 - T3 \right) \partial_z + \left(2 T1 x - 6 u T2 + 2 T2 y - 3 u T3 \right) \partial_u
\end{aligned}
\tag{2}$$

$$\begin{aligned}
&> e[0] := \text{evalDG}(\text{subs}(\{T1=0, T2=0, T3=0\}, LL)); \\
e_0 &:= 0 \partial_x + 0 \partial_y + 0 \partial_z + 0 \partial_u
\end{aligned}
\tag{3}$$

```

> e[1] := evalDG(subs({T1=1,T2=0,T3=0}, LL));
e[2] := evalDG(subs({T1=0,T2=1,T3=0}, LL));
e[3] := evalDG(subs({T1=0,T2=0,T3=1}, LL));

```

$$\begin{aligned}
e_1 &:= \partial_x + 0 \partial_y + 0 \partial_z + 2x \partial_u \\
e_2 &:= -3x \partial_x - (3y + z - 1) \partial_y - (y + 3z) \partial_z - (6u - 2y) \partial_u \\
e_3 &:= -\frac{3x}{2} \partial_x - 2y \partial_y - (y + z - 1) \partial_z - 3u \partial_u
\end{aligned}
\tag{4}$$

```

> algebre_lie := LieAlgebraData([seq(e[i], i=1..3)]);
DGsetup(algebre_lie);
LD := LeviDecomposition();
resoluble := Query("Solvable");
semi_simple := Query("Semisimple");
MultiplicationTable("LieTable");

```

$$\begin{aligned}
\text{algebre_lie} &:= [e1, e2] = -3e1, [e1, e3] = -\frac{3}{2}e1, [e2, e3] = -e2 + 2e3 \\
LD &:= [[e1, e2, e3], []] \\
\text{resoluble} &:= \text{true} \\
\text{semi_simple} &:= \text{false}
\end{aligned}$$

L1	$e1$	$e2$	$e3$
$e1$	0	$-3e1$	$-\frac{3}{2}e1$
$e2$	$3e1$	0	$-e2 + 2e3$
$e3$	$\frac{3}{2}e1$	$e2 - 2e3$	0

(5)

