

```
> restart:
```

```
with(PDEtools, casesplit, declare):
with(DEtools, gensys):
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```
with(DifferentialGeometry):
```

```
with(JetCalculus):
with(LieAlgebras):
with(GroupActions):
```

```
DGsetup([x,y,z,u], Rquatre):      Repere_xyzu := evalDG([D_x,
D_y,D_z,D_u]);
```

```
FF := x^2+y^2+x^3+F040*y^4+F130*x*y^3+F220*x^2*y^2+F310*x^3*y+
F400*x^4+(F220-(4/5)*F220*F400+(8/5)*F400^2-(3/5)*F400+(1/15)*
F310*F130-(1/5)*F310^2)*x^5+(-(9/8)*F310+(1/4)*F130-(1/3)*F400*
F130+3*F400*F310+(1/6)*F220*F130-(3/2)*F220*F310)*x^4*y+((1/3)*
F130^2-(4/3)*F310*F130-(4/3)*F220^2+(8/3)*F220*F400+F310^2-F220)*
x^3*y^2+(-(15/4)*F130+4*F400*F130-(7/3)*F220*F130+F220*F310+(2/3)
*F040*F130-2*F040*F310)*x^2*y^3+(-(1/3)*F130^2+F310*F130-9*F040
-4*F040*F220+8*F040*F400)*x*y^4+(-(22/15)*F040*F130+(2/15)*F220*
F130+(2/5)*F040*F310)*y^5;
```

$$\text{Repere_xyzu} := [\partial_x, \partial_y, \partial_z, \partial_u]$$

$$\begin{aligned} FF := & x^2 + y^2 + x^3 + F040 y^4 + F130 x y^3 + F220 x^2 y^2 + F310 x^3 y + F400 x^4 \\ & + \left(F220 - \frac{4}{5} F220 F400 + \frac{8}{5} F400^2 - \frac{3}{5} F400 + \frac{1}{15} F310 F130 \right. \\ & \left. - \frac{1}{5} F310^2 \right) x^5 + \left(-\frac{9}{8} F310 + \frac{1}{4} F130 - \frac{1}{3} F400 F130 + 3 F400 F310 \right. \\ & \left. + \frac{1}{6} F220 F130 - \frac{3}{2} F220 F310 \right) x^4 y + \left(\frac{1}{3} F130^2 - \frac{4}{3} F310 F130 - \frac{4}{3} F220^2 \right. \\ & \left. + \frac{8}{3} F220 F400 + F310^2 - F220 \right) x^3 y^2 + \left(-\frac{15}{4} F130 + 4 F400 F130 \right. \\ & \left. - \frac{7}{3} F220 F130 + F220 F310 + \frac{2}{3} F040 F130 - 2 F040 F310 \right) x^2 y^3 + \left(\right. \\ & \left. - \frac{1}{3} F130^2 + F310 F130 - 9 F040 - 4 F040 F220 + 8 F040 F400 \right) x y^4 + \left(\right. \end{aligned} \quad (1)$$

$$\left[-\frac{22}{15} F040 F130 + \frac{2}{15} F220 F130 + \frac{2}{5} F040 F310 \right] y^5$$

> e1 := evalDG((1+2*F220*x-4*F400*x+3*x+(1/3)*y*F130-y*F310-u*F220)*D_x-((1/3)*x*F130-x*F310-(9/2)*y-2*y*F220+4*y*F400+(1/2)*u*F130)*D_y+0*D_z+(4*F220*u-8*F400*u+9*u+2*x)*D_u);

e2 := evalDG((3*x*F130-x*F310+(4/3)*y*F040-(2/3)*y*F220-(3/2)*u*F130)*D_x-((4/3)*x*F040-1-(2/3)*F220*x-3*y*F130+y*F310+2*u*F040)*D_y+0*D_z+(6*F130*u-2*F310*u+2*y)*D_u);

e3 := evalDG(0*D_x+0*D_y+D_z+0*D_u);

e4 := evalDG(0*D_x+0*D_y+x*D_z+0*D_u);

e5 := evalDG(0*D_x+0*D_y+y*D_z+0*D_u);

e6 := evalDG(0*D_x+0*D_y+z*D_z+0*D_u);

e7 := evalDG(0*D_x+0*D_y+u*D_z+0*D_u);

$$e1 := \left(1 + 2 F220 x - 4 F400 x + 3 x + \frac{1}{3} y F130 - y F310 - u F220 \right) \partial_x - \left(\frac{1}{3} x F130 - x F310 - \frac{9}{2} y - 2 y F220 + 4 y F400 + \frac{1}{2} u F130 \right) \partial_y + \left(4 u F220 - 8 F400 u + 9 u + 2 x \right) \partial_u$$

$$e2 := \left(3 x F130 - x F310 + \frac{4}{3} y F040 - \frac{2}{3} y F220 - \frac{3}{2} u F130 \right) \partial_x - \left(\frac{4}{3} x F040 - 1 - \frac{2}{3} F220 x - 3 y F130 + y F310 + 2 u F040 \right) \partial_y + \left(6 u F130 - 2 F310 u + 2 y \right) \partial_u$$

$$e3 := \partial_z$$

$$e4 := x \partial_z$$

$$e5 := y \partial_z$$

$$e6 := z \partial_z$$

$$e7 := u \partial_z$$

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```
> B[1] := -32*F040*F310+4*F130*F220+136*F130*F400-20*F220*F310+24*
F310*F400-174*F130-27*F310;
```

```
B[2] := 8*F040*F130-24*F040*F310+88*F130*F400-16*F220*F310+24*
F310*F400-111*F130-27*F310;
```

```
B[3] := -4*F040*F220+8*F040*F400+F130^2+5*F130*F310-2*F220^2+4*
F220*F400-6*F040-3*F220;
```

```
B[4] := 8*F040^2-12*F040*F220+8*F040*F400+32*F130*F310-12*
F220^2+28*F220*F400+12*F040-21*F220;
```

$$B_1 := -32 F040 F310 + 4 F130 F220 + 136 F130 F400 - 20 F220 F310 \\ + 24 F310 F400 - 174 F130 - 27 F310$$

$$B_2 := 8 F040 F130 - 24 F040 F310 + 88 F130 F400 - 16 F220 F310 \\ + 24 F310 F400 - 111 F130 - 27 F310$$

$$B_3 := -4 F040 F220 + 8 F040 F400 + F130^2 + 5 F130 F310 - 2 F220^2 \\ + 4 F220 F400 - 6 F040 - 3 F220$$

$$B_4 := 8 F040^2 - 12 F040 F220 + 8 F040 F400 + 32 F130 F310 - 12 F220^2 \\ + 28 F220 F400 + 12 F040 - 21 F220 \quad (3)$$

```
> ## BASE DE GROEBNER COMPLETE
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```
with(Groebner):
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```
Ordre := tdeg:
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```
Variables := {F040, F130, F220, F310, F400}:
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```
VariablesOrdonnees := Ordre(seq(op(j,Variables),j=1..nops
(Variables))):
```

```
Ideal := {seq(B[i], i=1..4)}:
```

```
Hdim := HilbertDimension(Ideal);
```

```
Base := Basis(Ideal, VariablesOrdonnees):
```

```
NbBase := nops(Base);
```

```
for i from 1 to nops(Base) do
rB[i] := factor(op(i,Base))
```

od;

Ensemble_Base := {seq(op(i,Base), i=1..nops(Base))}:

Hdim:= 2

NbBase:= 11

$$rB_1 := -32 F040 F310 + 4 F130 F220 + 136 F130 F400 - 20 F220 F310 \\ + 24 F310 F400 - 174 F130 - 27 F310$$

$$rB_2 := -4 F040 F220 + 8 F040 F400 + F130^2 + 5 F130 F310 - 2 F220^2 \\ + 4 F220 F400 - 6 F040 - 3 F220$$

$$rB_3 := 8 F040 F130 - 24 F040 F310 + 88 F130 F400 - 16 F220 F310 \\ + 24 F310 F400 - 111 F130 - 27 F310$$

$$rB_4 := 8 F040^2 - 12 F040 F220 + 8 F040 F400 + 32 F130 F310 - 12 F220^2 \\ + 28 F220 F400 + 12 F040 - 21 F220$$

$$rB_5 := -229376 F040 F310 F400 + 8192 F130 F310^2 + 1032192 F130 F400^2 \\ + 1024 F220^2 F310 - 151552 F220 F310 F400 + 184320 F310 F400^2 \\ + 289680 F040 F310 - 2596608 F130 F400 + 189552 F220 F310 \\ - 434160 F310 F400 + 1632771 F130 + 255150 F310$$

$$rB_6 := 1024 F040 F220 F310 + 161792 F040 F310 F400 - 8192 F130 F310^2 \\ - 737280 F130 F400^2 + 105472 F220 F310 F400 - 129024 F310 F400^2 \\ - 205872 F040 F310 + 1854720 F130 F400 - 132816 F220 F310 \\ + 303696 F310 F400 - 1166265 F130 - 178362 F310$$

$$rB_7 := -1856 F040 F220 F400 + 320 F040 F310^2 + 3712 F040 F400^2 \\ + 1216 F130 F310 F400 + 16 F220^3 - 1056 F220^2 F400 + 208 F220 F310^2 \\ + 2048 F220 F400^2 - 288 F310^2 F400 + 2292 F040 F220 - 7368 F040 F400 \\ - 1452 F130 F310 + 1278 F220^2 - 4044 F220 F400 + 324 F310^2 + 3438 F040 \\ + 1881 F220$$

$$rB_8 := 32 F040 F220^2 + 2880 F040 F220 F400 - 1152 F040 F310^2 \\ - 5888 F040 F400^2 + 896 F130 F310 F400 + 1568 F220^2 F400 \\ - 736 F220 F310^2 - 3136 F220 F400^2 + 960 F310^2 F400 - 3636 F040 F220 \\ + 11784 F040 F400 - 1230 F130 F310 - 1950 F220^2 + 6252 F220 F400 \\ - 1080 F310^2 - 5526 F040 - 2925 F220$$

$$rB_9 := -12582912 F040 F310 F400^2 + 393216 F130 F310^2 F400 \\ + 56623104 F130 F400^3 + 32768 F220 F310^3 - 7864320 F220 F310 F400^2$$

$$\begin{aligned}
& -196608 F310^3 F400 + 9437184 F310 F400^3 + 38670336 F040 F310 F400 \\
& -801792 F130 F310^2 - 244629504 F130 F400^2 + 24293376 F220 F310 F400 \\
& + 221184 F310^3 - 40366080 F310 F400^2 - 28787184 F040 F310 \\
& + 346633344 F130 F400 - 18123696 F220 F310 + 55848528 F310 F400 \\
& - 161644329 F130 - 25178202 F310
\end{aligned}$$

$$rB_{10} := 589824 F040 F220 F400^2 - 557056 F040 F310^2 F400$$

$$\begin{aligned}
& -1179648 F040 F400^3 + 16384 F130 F310^3 + 1671168 F130 F310 F400^2 \\
& + 294912 F220^2 F400^2 - 352256 F220 F310^2 F400 - 589824 F220 F400^3 \\
& + 442368 F310^2 F400^2 - 1483776 F040 F220 F400 + 706080 F040 F310^2 \\
& + 3852288 F040 F400^2 - 4220928 F130 F310 F400 - 741888 F220^2 F400 \\
& + 444768 F220 F310^2 + 1926144 F220 F400^2 - 1048032 F310^2 F400 \\
& + 933012 F040 F220 - 4091688 F040 F400 + 2665170 F130 F310 \\
& + 466506 F220^2 - 2045844 F220 F400 + 619164 F310^2 + 1399518 F040 \\
& + 699759 F220
\end{aligned}$$

$$rB_{11} := 131072 F040 F310^3 + 50331648 F040 F310 F400^2$$

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$$\begin{aligned}
& -2621440 F130 F310^2 F400 - 226492416 F130 F400^3 \\
& + 31457280 F220 F310 F400^2 + 393216 F310^3 F400 - 37748736 F310 F400^3 \\
& - 145059840 F040 F310 F400 + 4094976 F130 F310^2 \\
& + 934723584 F130 F400^2 - 90869760 F220 F310 F400 - 442368 F310^3 \\
& + 152506368 F310 F400^2 + 102967632 F040 F310 - 1276363008 F130 F400 \\
& + 64551600 F220 F310 - 202200624 F310 F400 + 577301175 F130 \\
& + 88207542 F310
\end{aligned}$$

```

> Tableau_crochets := Matrix(7, 7, {(1, 1) = 0, (1, 2) = (8/3)*
F130*ee1+(-9/2-(4/3)*F220+4*F400-(4/3)*F040)*ee2, (1, 3) = 0, (1,
4) = ee3+(2*F220-4*F400+3)*ee4+((1/3)*F130-F310)*ee5-F220*ee7,
(1, 5) = -(1/3)*F130+F310)*ee4+(9/2+2*F220-4*F400)*ee5-(1/2)*
F130*ee7, (1, 6) = 0, (1, 7) = 2*ee4+(4*F220-8*F400+9)*ee7, (2,
1) = -(8/3)*F130*ee1-(-9/2-(4/3)*F220+4*F400-(4/3)*F040)*ee2, (2,
2) = 0, (2, 3) = 0, (2, 4) = (3*F130-F310)*ee4+((4/3)*F040-(2/3)*
F220)*ee5-(3/2)*F130*ee7, (2, 5) = ee3+(-(4/3)*F040+(2/3)*F220)*
ee4+(3*F130-F310)*ee5-2*F040*ee7, (2, 6) = 0, (2, 7) = 2*ee5+(6*
F130-2*F310)*ee7, (3, 1) = 0, (3, 2) = 0, (3, 3) = 0, (3, 4) = 0,
(3, 5) = 0, (3, 6) = ee3, (3, 7) = 0, (4, 1) = -ee3-(2*F220-4*
F400+3)*ee4-((1/3)*F130-F310)*ee5+F220*ee7, (4, 2) = -(3*F130-
F310)*ee4-((4/3)*F040-(2/3)*F220)*ee5+(3/2)*F130*ee7, (4, 3) = 0,
(4, 4) = 0, (4, 5) = 0, (4, 6) = ee4, (4, 7) = 0, (5, 1) = -(-

```

$$(1/3)*F130+F310)*ee4-(9/2+2*F220-4*F400)*ee5+(1/2)*F130*ee7, (5, 2) = -ee3-(-(4/3)*F040+(2/3)*F220)*ee4-(3*F130-F310)*ee5+2*F040*ee7, (5, 3) = 0, (5, 4) = 0, (5, 5) = 0, (5, 6) = ee5, (5, 7) = 0, (6, 1) = 0, (6, 2) = 0, (6, 3) = -ee3, (6, 4) = -ee4, (6, 5) = -ee5, (6, 6) = 0, (6, 7) = -ee7, (7, 1) = -2*ee4-(4*F220-8*F400+9)*ee7, (7, 2) = -2*ee5-(6*F130-2*F310)*ee7, (7, 3) = 0, (7, 4) = 0, (7, 5) = 0, (7, 6) = ee7, (7, 7) = 0));$$

$$Tableau_crochets:= \left[\left[0, \frac{8}{3} F130 ee1 + \left(-\frac{9}{2} - \frac{4}{3} F220 + 4 F400 - \frac{4}{3} F040 \right) ee2, \quad (5) \right. \right.$$

$$0, ee3 + (2 F220 - 4 F400 + 3) ee4 + \left(\frac{1}{3} F130 - F310 \right) ee5 - F220 ee7, \left(-\frac{1}{3} F130 + F310 \right) ee4 + \left(\frac{9}{2} + 2 F220 - 4 F400 \right) ee5 - \frac{1}{2} F130 ee7, 0, 2 ee4 + (4 F220 - 8 F400 + 9) ee7 \right],$$

$$\left[-\frac{8}{3} F130 ee1 - \left(-\frac{9}{2} - \frac{4}{3} F220 + 4 F400 - \frac{4}{3} F040 \right) ee2, 0, 0, (3 F130 - F310) ee4 + \left(\frac{4}{3} F040 - \frac{2}{3} F220 \right) ee5 - \frac{3}{2} F130 ee7, ee3 + \left(-\frac{4}{3} F040 + \frac{2}{3} F220 \right) ee4 + (3 F130 - F310) ee5 - 2 F040 ee7, 0, 2 ee5 + (6 F130 - 2 F310) ee7 \right],$$

$$\left[0, 0, 0, 0, 0, ee3, 0 \right],$$

$$\left[-ee3 - (2 F220 - 4 F400 + 3) ee4 - \left(\frac{1}{3} F130 - F310 \right) ee5 + F220 ee7, - (3 F130 - F310) ee4 - \left(\frac{4}{3} F040 - \frac{2}{3} F220 \right) ee5 + \frac{3}{2} F130 ee7, 0, 0, 0, ee4, 0 \right],$$

$$\left[-\left(-\frac{1}{3} F130 + F310 \right) ee4 - \left(\frac{9}{2} + 2 F220 - 4 F400 \right) ee5 + \frac{1}{2} F130 ee7, -ee3 - \left(-\frac{4}{3} F040 + \frac{2}{3} F220 \right) ee4 - (3 F130 - F310) ee5 + 2 F040 ee7, 0, 0, 0, ee5, 0 \right],$$

$$\left[0, 0, -ee3, -ee4, -ee5, 0, -ee7 \right],$$

$$\left[\begin{array}{l} -2\,ee4 - (4\,F220 - 8\,F400 + 9)\,ee7, -2\,ee5 - (6\,F130 - 2\,F310)\,ee7, 0, 0, 0, \\ ee7, 0 \end{array} \right]$$