

```
> restart:
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```
with(PDEtools, casesplit, declare):  
with(DEtools, gensys):
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with(DifferentialGeometry):
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with(JetCalculus):  
with(LieAlgebras):  
with(GroupActions):
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```
DGsetup([x,y,z,u], Rquatre):      Repere_xyzu := evalDG([D_x,  
D_y,D_z,D_u]);
```

```
XX := (-4*F400*T1-T1)*x+T2*y-2*F400*T1*u+T1;
```

```
YY := -T2*x+(-(1/2)*T3-4*F400*T1-T1)*y-T2*z+T2;
```

```
ZZ := (4*F400*T1-T3)*x-T2*y+(-T1-T3)*z+2*F400*T1*u+T3;
```

```
UU := 2*T1*x+2*T2*y+(-8*F400*T1-2*T1)*u;
```

```
FF := sort(expand(
```

```
(1/2)*x*y^4+(1/2)*y^4*z+(3/4)*y^4*z^2+x^2+y^2+(1/4)*y^4+y^2*z+  
F400*x^4+x*y^2*z+y^2*z^2+((2/5)*F400+(8/5)*F400^2)*x^5+x^2*y^2*z+  
y^2*z^3+((18/5)*F400^2+(16/5)*F400^3+(1/5)*F400)*x^6+(-4*F400^2  
-3*F400)*x^4*y^2+((3/2)*F400+3/4)*x^2*y^4+(-2*F400+1)*x^3*y^2*z+  
y^2*z^4+((116/35)*F400^2+(464/35)*F400^3+(256/35)*F400^4+(4/35)*  
F400)*x^7+(-(48/5)*F400^3-(74/5)*F400^2-(18/5)*F400)*x^5*y^2+(4*  
F400^2+4*F400+1)*x^3*y^4+(-4*F400^2-5*F400+1)*x^4*y^2*z+(3*  
F400+3)*x^2*y^4*z+(-2*F400+4)*x^3*y^2*z^2+y^4*z^3+y^2*z^5-2*F400*  
x^3*y^2+2*x*y^2*z^2+(3/2)*x*y^4*z+3*x^2*y^2*z^2+3*x*y^2*z^3-(1/2)  
*F400*x*y^6+3*x*y^4*z^2+6*x^2*y^2*z^3+4*x*y^2*z^4
```

```
), [z,y,x], ascending);
```

```
indets({XX,YY,ZZ,UU}) minus {T1,T2,T3,x,y,z,u};
```

$Repere_xyzu := [\partial_x, \partial_y, \partial_z, \partial_u]$

$$XX := (-4 F400 T1 - T1) x + T2 y - 2 F400 T1 u + T1$$

$$YY := -T2 x + \left(-\frac{1}{2} T3 - 4 F400 T1 - T1 \right) y - T2 z + T2$$

$$ZZ := (4 F400 T1 - T3) x - T2 y + (-T1 - T3) z + 2 F400 T1 u + T3$$

$$UU := 2 T1 x + 2 T2 y + (-8 F400 T1 - 2 T1) u$$

$$\begin{aligned} FF := & x^2 + y^2 + z y^2 + F400 x^4 + \frac{1}{4} y^4 + z y^2 x + z^2 y^2 + \frac{2}{5} F400 x^5 + \frac{8}{5} F400^2 x^5 \\ & - 2 F400 y^2 x^3 + \frac{1}{2} y^4 x + z y^2 x^2 + \frac{1}{2} z y^4 + 2 z^2 y^2 x + z^3 y^2 + \frac{18}{5} F400^2 x^6 \\ & + \frac{16}{5} F400^3 x^6 + \frac{1}{5} F400 x^6 - 4 F400^2 y^2 x^4 - 3 F400 y^2 x^4 + \frac{3}{4} y^4 x^2 \\ & + \frac{3}{2} F400 y^4 x^2 + z y^2 x^3 - 2 F400 z y^2 x^3 + \frac{3}{2} z y^4 x + 3 z^2 y^2 x^2 + \frac{3}{4} z^2 y^4 \\ & + 3 z^3 y^2 x + z^4 y^2 + \frac{116}{35} F400^2 x^7 + \frac{464}{35} F400^3 x^7 + \frac{256}{35} F400^4 x^7 \\ & + \frac{4}{35} F400 x^7 - \frac{48}{5} F400^3 y^2 x^5 - \frac{74}{5} F400^2 y^2 x^5 - \frac{18}{5} F400 y^2 x^5 + y^4 x^3 \\ & + 4 F400^2 y^4 x^3 + 4 F400 y^4 x^3 - \frac{1}{2} F400 y^6 x + z y^2 x^4 - 4 F400^2 z y^2 x^4 \\ & - 5 F400 z y^2 x^4 + 3 z y^4 x^2 + 3 F400 z y^4 x^2 + 4 z^2 y^2 x^3 - 2 F400 z^2 y^2 x^3 \\ & + 3 z^2 y^4 x + 6 z^3 y^2 x^2 + z^3 y^4 + 4 z^4 y^2 x + z^5 y^2 \\ & \{F400\} \end{aligned}$$

(1)

```
> LL := evalDG(
  XX*D_x
+
  YY*D_y
+
  ZZ*D_z
+
  UU*D_u
) ;
```

Composantes_LL := GetComponent(LL, Repere_xyzu):

$$LL := - \left(2 F400 T1 u + 4 F400 T1 x + T1 x - T2 y - T1 \right) \partial_x - \left(T2 x + \frac{1}{2} y T3 + 4 y F400 T1 + y T1 + T2 z - T2 \right) \partial_y + \left(2 F400 T1 u + 4 F400 T1 x - z T1 - T2 y - T3 x - z T3 + T3 \right) \partial_z - \left(8 F400 T1 u + 2 T1 u - 2 T1 x - 2 T2 y \right) \partial_u \quad (2)$$

> e[0] := evalDG(subs({T1=0,T2=0,T3=0}, LL));

$$e_0 := 0 \partial_x + 0 \partial_y + 0 \partial_z + 0 \partial_u \quad (3)$$

**> e[1] := evalDG(subs({T1=1,T2=0,T3=0}, LL));
e[2] := evalDG(subs({T1=0,T2=1,T3=0}, LL);
e[3] := evalDG(subs({T1=0,T2=0,T3=1}, LL));**

$$e_1 := - \left(2 F400 u + 4 F400 x + x - 1 \right) \partial_x - \left(4 y F400 + y \right) \partial_y + \left(2 F400 u + 4 F400 x - z \right) \partial_z - \left(8 F400 u + 2 u - 2 x \right) \partial_u$$

$$e_2 := y \partial_x - \left(x - 1 + z \right) \partial_y - y \partial_z + 2 y \partial_u$$

$$e_3 := 0 \partial_x - \frac{y}{2} \partial_y - \left(x - 1 + z \right) \partial_z + 0 \partial_u \quad (4)$$

> algebre_lie := LieAlgebraData([seq(e[i], i=1..3)]);

DGsetup(algebre_lie);

LD := LeviDecomposition();

resoluble := Query("Solvable");

semi_simple := Query("Semisimple");

MultiplicationTable("LieTable");

$$algebre_lie := [e1, e2] = 4 F400 e2, [e1, e3] = 0, \left[e2, e3 \right] = \frac{1}{2} e2$$

$$LD := [[e1, e2, e3], [\]]$$

$$resoluble := true$$

$$semi_simple := false$$

L1	e1	e2	e3
e1	0	4 F400 e2	0
e2	- 4 F400 e2	0	$\frac{1}{2} e2$
e3	0	$-\frac{1}{2} e2$	0

(5)