Key Vanishing Theorems

1. Logarithmic Jet Differentials

On $\mathbb{P}^2 = \mathbb{P}^2(\mathbb{C})$, let $[T\colon X\colon Y]$ be homogeneous coordinates. Consider 3 homogeneous polynomials:

$$A(T,X,Y), \qquad\qquad B(T,X,Y), \qquad\qquad C(T,X,Y),$$

of respective degrees $d_A, d_B, d_C \geqslant 1$. Later, A, B, C will be of equal degree 2, with zero-sets defining three conic curves of $\mathbb{P}^2$, mutually in general position.

For now, let us assume more generally that:

$$1 \leqslant d_A = d_B = d_C =: d_{ABC},$$

and let us point out that inequal degrees could equally be treated after some mild adaptations.

Also, let us restrict ourselves to jets of order 2, with derivatives of (entire) holomorphic curves:

$$\zeta \longmapsto \big(T(\zeta),\, X(\zeta),\, Y(\zeta)\big),$$

having corresponding jet coordinates denoted as:

$$T',\, X',\, Y',\; T'',\, X'',\, Y''.$$

As is known, with the coordinate axes divisor:

$$\mathscr{H} := \big\{T=0\big\} \cup \big\{X=0\big\} \cup \big\{Y=0\big\},$$

for any weighted degree $m \geqslant 1$, the logarithmic 2-jet differential bundle $E_{2,m}T^*_{\mathbb{P}^2}\big(\log \mathscr{H}\big)$ of weight m is freely and globally generated by the following sections:

$$\Big(\big(\log \tfrac{X}{T}\big)'\Big)^i \Big(\big(\log \tfrac{Y}{T}\big)'\Big)^j \begin{vmatrix} \big(\log \frac{X}{T}\big)' & \big(\log \frac{Y}{T}\big)' \\ \big(\log \frac{X}{T}\big)'' & \big(\log \frac{Y}{T}\big)'' \end{vmatrix}^k,$$

for all integers $i, j, k \geqslant 0$ satisfying:

$$i + j + 3\,k = m.$$

More simply, the logarithmic 1-jet differential bundle $E_{1,m}T^*_{\mathbb{P}^2}\big(\log \mathscr{H}\big)$ has the generators:

$$\Big(\big(\log \tfrac{X}{T}\big)'\Big)^i \Big(\big(\log \tfrac{Y}{T}\big)'\Big)^j \qquad\qquad {\scriptstyle (i+j\,=\,m)}.$$

Of course, thanks to $\log \frac{X}{T} = -\log \frac{T}{X}$, *etc.*, the roles of T, X, Y can be permuted.

On the affine chart $\big\{T \neq 0\big\}$ of $\mathbb{P}^2$, denote homogeneous coordinates as:

$$(x,y) \qquad\qquad \text{where} \qquad\qquad x := \tfrac{X}{T}, \qquad y := \tfrac{Y}{T},$$

and set:

$$a(x,y) := A(1,x,y), \qquad\qquad b(x,y) := B(1,x,y), \qquad\qquad c(x,y) := C(1,x,y),$$

Consider the *Jacobian* of the 3 polynomials A, B, C of equal degrees $d_A = d_B = d_C$:

$$
\begin{aligned}
J \,:=\, & \begin{vmatrix} A_T & B_T & C_T \\ A_X & B_X & C_X \\ A_Y & B_Y & C_Y \end{vmatrix} \\
& = A_T\,B_X\,C_Y + A_X\,B_Y\,C_T + A_Y\,B_T\,C_X - A_Y\,B_X\,C_T - A_X\,B_T\,C_Y - A_T\,B_Y\,C_X.
\end{aligned}
$$

By homogeneity:

$$
\begin{aligned}
A(T,X,Y) \,&=\, T^{d_A}\,A\big(1,\tfrac{X}{T},\tfrac{Y}{T}\big) \,=\, T^{d_A}\,A(1,x,y),\\
B(T,X,Y) \,&=\, T^{d_B}\,B\big(1,\tfrac{X}{T},\tfrac{Y}{T}\big) \,=\, T^{d_B}\,B(1,x,y),\\
C(T,X,Y) \,&=\, T^{d_C}\,C\big(1,\tfrac{X}{T},\tfrac{Y}{T}\big) \,=\, T^{d_C}\,C(1,x,y),
\end{aligned}
$$

that is:

$$
A(T,xT,yT) \,=\, T^{d_A}\,a(x,y), \qquad B(T,xT,yT) \,=\, T^{d_B}\,b(x,y), \qquad C(T,xT,yT) \,=\, T^{d_C}\,c(x,y),
$$

whence by differentiation with respect to T, to x, to y:

$$
\begin{aligned}
A_T \,&=\, d_A\,T^{d_A-1}\,a & B_T \,&=\, d_B\,T^{d_B-1}\,b & C_T \,&=\, d_C\,T^{d_C-1}\,c\\
TA_X \,&=\, T^{d_A}\,a_x, & TB_X \,&=\, T^{d_B}\,b_x, & TB_X \,&=\, T^{d_C}\,c_x,\\
TA_Y \,&=\, T^{d_A}\,a_y & TB_Y \,&=\, T^{d_B}\,b_y & TC_Y \,&=\, T^{d_C}\,c_y
\end{aligned}
$$

and hence:

$$
\begin{aligned}
J \,&=\, \begin{vmatrix} d_AT^{d_A-1}a & d_BT^{d_B-1}b & d_CT^{d_C-1}\,c \\ T^{d_A-1}a_x & T^{d_B-1}b_x & T^{d_C-1}c_x \\ T^{d_A-1}a_y & T^{d_B-1}b_y & T^{d_C-1}c_y \end{vmatrix}\\
&=\, d_{ABC}\,T^{d_A-1+d_B-1+d_C-1}\begin{vmatrix} a & b & c \\ a_x & b_x & c_x \\ a_y & b_y & c_y \end{vmatrix} \,=:\, D.
\end{aligned}
$$

In the case of three conics:

$$
J \,=\, 2\,T^3\,D.
$$

Consider the divisor:

$$
\mathscr{D} \,:=\, \big\{A=0\big\}\cup\big\{B=0\big\}\cup\big\{C=0\big\},
$$

and assume that the coefficients of the three homogeneous polynomials A, of B, of C are generic enough to insure that:

- $\big\{J\,=\,0\big\}$ has finite (algebraically transverse) intersection in $\mathbb{P}^2$ with each one of the 3 coordinate lines $\big\{T=0\big\}$, $\big\{X=0\big\}$, $\big\{Y=0\big\}$;
- $\big\{J\,=\,0\big\}$ has finite (algebraically transverse) intersection in $\mathbb{P}^2$ with each one of the 3 algebraic curves $\big\{A=0\big\}$, $\big\{B=0\big\}$, $\big\{C=0\big\}$.

Then according to a previously established proposition, on the Zariski-open complement:

$$
\mathbb{P}^2\backslash\big\{J=0\big\},
$$

the logarithmic 2-jet differential bundle $E_{2,m}T^*_{\mathbb{P}^2}\big(\log\mathscr{D}\big)$ is freely generated by the sections:

$$
\Big(\big(\log\tfrac{A}{C}\big)'\Big)^i\,\Big(\big(\log\tfrac{B}{C}\big)'\Big)^j\,\begin{vmatrix}\big(\log\frac{A}{C}\big)' & \big(\log\frac{B}{C}\big)' \\ \big(\log\frac{A}{C}\big)'' & \big(\log\frac{B}{C}\big)''\end{vmatrix}^k,
$$

for all integers $i, j, k \geqslant 0$ satisfying:

$$i + j + 3\,k \,=\, m.$$

More simply, the logarithmic 1-jet differential bundle $E_{1,m}T^*_{\mathbb{P}^2}\big(\mathsf{log}\,\mathscr{D}\big)$ has the generators:

$$\Big(\big(\mathsf{log}\,\tfrac{A}{C}\big)'\Big)^i\,\Big(\big(\mathsf{log}\,\tfrac{B}{C}\big)'\Big)^j \qquad\qquad {\scriptstyle (i+j\,=\,m)}.$$

Here by equality of degrees and by homogeneity:

$$\frac{A(T,X,Y)}{C(T,X,Y)} \,=\, \frac{T^{d_A}\,A(1,x,y)}{T^{d_C}\,C(1,x,y)} \,=\, \frac{a(x,y)}{c(x,y)} \qquad\qquad {\scriptstyle (\text{in chart } \{T \neq 0\})},$$

and the same holds for $\frac{B}{C}$. By jet differentiation of holomorphic curves $\zeta \longmapsto \big(x(\zeta), y(\zeta)\big)$ valued in the affine chart $\mathbb{C}^2 \ni (x,y)$, it follows that:

$$\begin{aligned}
\big(\mathsf{log}\,\tfrac{A}{C}\big)' \,&=\, \big(\mathsf{log}\,\tfrac{a}{c}\big)' \,=\, \big(\mathsf{log}\,a\big)' - \big(\mathsf{log}\,c\big)' \,=\, \frac{a'}{a} - \frac{c'}{c},\\
\big(\mathsf{log}\,\tfrac{A}{C}\big)'' \,&=\, \big(\mathsf{log}\,\tfrac{a}{c}\big)'' \,=\, \big(\mathsf{log}\,a\big)'' - \big(\mathsf{log}\,c\big)'' \,=\, \frac{a''}{a} - \frac{a'a'}{a\,a} - \frac{c''}{c} + \frac{c'c'}{c\,c},
\end{aligned}$$

with, similarly:

$$\begin{aligned}
\big(\mathsf{log}\,\tfrac{B}{C}\big)' \,&=\, \frac{b'}{b} - \frac{c'}{c},\\
\big(\mathsf{log}\,\tfrac{B}{C}\big)'' \,&=\, \frac{b''}{b} - \frac{b'b'}{b\,b} - \frac{c''}{c} + \frac{c'c'}{c\,c},
\end{aligned}$$

where:

$$\begin{aligned}
a' \,&:=\, x'\,a_x + y'\,a_y, & a'' \,&:=\, x''\,a_x + y''\,a_y + x'x'\,a_{xx} + 2\,x'y'\,a_{xy} + y'y'\,a_{yy},\\
b' \,&:=\, x'\,b_x + y'\,b_y, & b'' \,&:=\, x''\,b_x + y''\,b_y + x'x'\,b_{xx} + 2\,x'y'\,b_{xy} + y'y'\,b_{yy},\\
c' \,&:=\, x'\,c_x + y'\,c_y, & c'' \,&:=\, x''\,c_x + y''\,c_y + x'x'\,c_{xx} + 2\,x'y'\,c_{xy} + y'y'\,c_{yy}.
\end{aligned}$$

Consequently:

$$\begin{aligned}
\overline{x}' \,&:=\, \Big(\mathsf{log}\,\frac{a}{c}\Big)' \,=\, \frac{x'a_x + y'a_y}{a} - \frac{x'c_x + y'c_y}{c} \,=\, \underbrace{\frac{a_x\,c - a\,c_x}{a\,c}}_{=:\,\alpha}\,x' + \underbrace{\frac{a_y\,c - a\,c_y}{a\,c}}_{=:\,\beta}\,y',\\
\overline{y}' \,&:=\, \Big(\mathsf{log}\,\frac{b}{c}\Big)' \,=\, \frac{x'b_x + y'b_y}{b} - \frac{x'c_x + y'c_y}{c} \,=\, \underbrace{\frac{b_x\,c - b\,c_x}{b\,c}}_{=:\,\delta}\,x' + \underbrace{\frac{b_y\,c - b\,c_y}{b\,c}}_{=:\,\gamma}\,y'.
\end{aligned}$$

In what follows, we will abbreviate these 1[st] jet transfer formulas from standard jets (x', y') to logarithmic jets $(\overline{x}', \overline{y}')$ as:

$$\begin{aligned}
\overline{x}' \,&=\, \alpha\,x' + \beta\,y', & \alpha \,&:=\, \tfrac{a_x\,c - a\,c_x}{a\,c}, & \beta \,&:=\, \tfrac{a_y\,c - a\,c_y}{a\,c},\\
\overline{y}' \,&=\, \gamma\,x' + \delta\,y'. & \beta \,&:=\, \tfrac{b_x\,c - b\,c_x}{b\,c}, & \delta \,&:=\, \tfrac{b_y\,c - b\,c_y}{b\,c}.
\end{aligned}$$

Because logarithmic jet differentials are by definition assumed to exist and to be holomorphic outside the considered divisor — here $\mathscr{D} = \big\{A\,B\,C = 0\big\}$ which may be three conics —, the presence of a, b, c (within the affine $\mathbb{C}^2 \subset \mathbb{P}^2$) in denominator places does not produce any singularity.

The only possible singularity — which enlightens why the proposition mentioned above holds in the complement $\mathbb{P}^2 \backslash \{J = 0\}$ — comes from inverting the transfer formulas from logarithmic jets to standard jets:

$$\begin{aligned} \tfrac{\delta}{\alpha\,\delta - \beta\,\gamma}\,\overline{x}' - \tfrac{\beta}{\alpha\,\delta - \beta\,\gamma}\,\overline{x}' &= x', \\ -\tfrac{\gamma}{\alpha\,\delta - \beta\,\gamma}\,\overline{x}' + \tfrac{\alpha}{\alpha\,\delta - \beta\,\gamma}\,\overline{x}' &= y', \end{aligned}$$

and this requires the nonvanishing of the 2×2 determinant:

$$\begin{vmatrix} \alpha & \beta \\ \gamma & \delta \end{vmatrix} = \begin{vmatrix} \frac{a_x c - a c_x}{a\,c} & \frac{a_y c - a c_y}{a\,c} \\ \frac{b_x c - b c_x}{b\,c} & \frac{b_y c - b c_y}{b\,c} \end{vmatrix} = \begin{vmatrix} a & b & c \\ a_x & b_x & c_x \\ a_y & b_y & c_y \end{vmatrix} = D = \frac{J}{2\,T^3},$$

which happens, by a direct computation in the affine chart $\{T \neq 0\}$, to be a nonzero multiple of the Jacobian determinant J.

The transfer of the logarithmic Wronskian, from

$$\overline{W}''' = \begin{vmatrix} \big(\log \frac{A}{C}\big)' & \big(\log \frac{B}{C}\big)' \\ \big(\log \frac{A}{C}\big)'' & \big(\log \frac{B}{C}\big)'' \end{vmatrix} = \begin{vmatrix} \big(\log \frac{a}{c}\big)' & \big(\log \frac{b}{c}\big)' \\ \big(\log \frac{a}{c}\big)'' & \big(\log \frac{b}{c}\big)'' \end{vmatrix}$$

is much more delicate, computationally, as it requires to expand:

$$\overline{W}''' = \begin{vmatrix} \alpha\,x' + \beta\,y' & \gamma\,x' + \delta\,y' \\ \alpha\,x'' + \beta\,y'' + \cdots & \gamma\,x'' + \delta\,y'' + \cdots \end{vmatrix},$$

the cdots being the already not small 'remainders' in:

$$\begin{aligned} \Big(\log \frac{a}{c}\Big)'' = {} & x'' \Big(\frac{a_x\,c - a\,c_x}{a\,c}\Big) + y'' \Big(\frac{a_y\,c - a\,c_y}{a\,c}\Big) \\ & + x'x' \Big(\frac{a_{xx}}{a} - \frac{a_x^2}{a^2} - \frac{c_{xx}}{c} + \frac{c_x^2}{c^2}\Big) \\ & + 2\,x'y' \Big(\frac{a_{xy}}{a} - \frac{a_x\,a_y}{a^2} - \frac{c_{xy}}{c} + \frac{c_x\,c_y}{c^2}\Big) \\ & + 2\,x'y' \Big(\frac{a_{yy}}{a} - \frac{a_y^2}{a^2} - \frac{c_{yy}}{c} + \frac{c_y^2}{c^2}\Big), \end{aligned}$$

with a similar formula for $\left(\log\frac{b}{c}\right)''$. Then a full expansion of the logarithmic Wronskian is:

$$
\begin{aligned}
\overline{W}''' \,&=\, \big(\alpha\,\gamma-\beta\,\delta\big)\begin{vmatrix} x' & y' \\ x'' & y'' \end{vmatrix} \\
&+ x'x'x'\bigg(\frac{a_x\,b_{xx}}{a\,b}-\frac{a_x\,c_{xx}}{a\,c}-\frac{c_x\,b_{xx}}{c\,b}-\frac{b_x\,a_{xx}}{b\,a}+\frac{c_x\,a_{xx}}{c\,a}+\frac{b_x\,c_{xx}}{b\,c} \\
&\qquad\qquad -\frac{a_x\,b_x^2}{a\,b^2}+\frac{a_x\,c_x^2}{a\,c^2}+\frac{b_x^2\,c_x}{b^2\,c}+\frac{a_x^2\,b_x}{a^2\,b}-\frac{a_x^2\,c_x}{a^2\,c}-\frac{b_x\,c_x^2}{b\,c^2}\bigg) \\
&+ x'x'y'\bigg(2\,\frac{a_x\,b_{xy}}{a\,b}-2\,\frac{a_x\,c_{xy}}{a\,c}+\frac{a_y\,b_{xx}}{a\,b}-\frac{a_y\,c_{xx}}{a\,c}-2\,\frac{c_x\,b_{xy}}{c\,b}-\frac{c_y\,b_{xx}}{c\,b} \\
&\qquad\qquad -\frac{b_y\,a_{xx}}{b\,a}+\frac{c_y\,a_{xx}}{c\,a}-2\,\frac{b_x\,a_{xy}}{b\,a}+2\,\frac{c_x\,a_{xy}}{c\,a}+\frac{b_y\,c_{xx}}{b\,c}+2\,\frac{b_x\,c_{xy}}{b\,c} \\
&\qquad\qquad -2\,\frac{a_x\,b_x\,b_y}{a\,b\,b}+2\,\frac{a_x\,c_x\,c_y}{a\,c\,c}+2\,\frac{b_x\,b_y\,c_x}{b\,b\,c}+2\,\frac{a_x\,a_y\,b_x}{a\,a\,b}-2\,\frac{a_x\,a_y\,c_x}{a\,a\,c}-2\,\frac{b_x\,c_x\,c_y}{b\,c\,c} \\
&\qquad\qquad -\frac{a_y\,b_x\,b_x}{a\,b\,b}+\frac{a_y\,c_x\,c_x}{a\,c\,c}+\frac{b_x\,b_x\,c}{b\,b\,c}+\frac{a_x\,a_x\,b_y}{a\,a\,b}-\frac{a_x\,a_x\,c_y}{a\,a\,c}-\frac{b_y\,c_x\,c_x}{b\,c\,c}\bigg) \\
&+ x'y'y'\bigg(\frac{a_x\,b_{yy}}{a\,b}-\frac{a_x\,c_{yy}}{a\,c}+2\,\frac{a_y\,b_{xy}}{a\,b}-2\,\frac{a_y\,c_{xy}}{a\,c}-\frac{c_x\,b_{yy}}{c\,b}-2\,\frac{c_y\,b_{xy}}{c\,b}- \\
&\qquad\qquad -2\,\frac{b_y\,a_{xy}}{a}+2\,\frac{c_y\,a_{xy}}{c\,b}-\frac{b_x\,a_{yy}}{b\,a}+\frac{c_x\,a_{yy}}{c\,a}+2\,\frac{b_y\,c_{xy}}{b\,c}+\frac{b_x\,c_{yy}}{b\,c} \\
&\qquad\qquad -\frac{a_x\,b_y\,b_y}{a\,b\,b}-2\,\frac{a_y\,b_x\,b_y}{a\,b\,b}+2\,\frac{a_y\,c_x\,c_y}{a\,c\,c}+2\,\frac{b_x\,b_y\,c_y}{b\,b\,c}+2\,\frac{a_x\,a_y\,b_y}{a\,a\,b}-2\,\frac{a_x\,a_y\,c_y}{a\,a\,c} \\
&\qquad\qquad -2\,\frac{b_y\,c_x\,c_y}{b\,c\,c}+\frac{a_x\,c_y\,c_y}{a\,c\,c}+\frac{b_y\,b_y\,c_x}{b\,b\,c}+\frac{a_y\,a_y\,b_x}{a\,a\,b}-\frac{a_y\,a_y\,c_x}{a\,a\,c}-\frac{b_x\,c_y\,c_y}{b\,c\,c}\bigg) \\
&+ y'y'y'\bigg(\frac{a_y\,b_{yy}}{a\,b}-\frac{a_y\,c_{yy}}{a\,c}-\frac{c_y\,b_{yy}}{c\,b}-\frac{b_y\,a_{yy}}{b\,a}+\frac{c_y\,a_{yy}}{c\,a}+\frac{b_y\,c_{yy}}{b\,c} \\
&\qquad\qquad -\frac{a_y\,b_y\,b_y}{a\,b\,b}+\frac{a_y\,c_y\,c_y}{a\,c\,c}+\frac{b_y\,b_y\,c_y}{b\,b\,c}+\frac{a_y\,a_y\,b_y}{a\,a\,b}-\frac{a_y\,a_y\,c_y}{a\,a\,c}-\frac{b_y\,c_y\,c_y}{b\,c\,c}\bigg).
\end{aligned}
$$

Key Observation 1.1. *In the logarithmic Wronskian transferred to the standard affine jet coordinates (x', y', x'', y''):*

$$
\begin{aligned}
\overline{W}''' \,&=\, \big(\alpha\,\gamma-\beta\,\delta\big)\begin{vmatrix} x' & y' \\ x'' & y'' \end{vmatrix} \\
&\qquad + \kappa\,{x'}^3+\lambda\,{x'}^2y'+\mu\,x'{y'}^2+\nu\,{y'}^3,
\end{aligned}
$$

the remainder cubic terms in the first order jets x', y' have coefficients κ, λ, μ, ν which express in terms of the second order derivatives:

$$
a_{xx},\ \ a_{xy},\ \ a_{yy}, \qquad b_{xx},\ \ b_{xy},\ \ b_{yy}, \qquad c_{xx},\ \ c_{xy},\ \ c_{yy},
$$

and in terms of α, β, γ, δ as follows:

$$
\begin{aligned}
\kappa \ :=\ & -\gamma\,\frac{a_{xx}}{a} + \alpha\,\frac{b_{xx}}{b} - (\alpha-\gamma)\,\frac{c_{xx}}{c} \\
& + \alpha^2\,\gamma - \alpha\,\gamma^2, \\
\lambda \ :=\ & -\delta\,\frac{a_{xx}}{a} - 2\,\gamma\,\frac{a_{xy}}{a} + \beta\,\frac{b_{xx}}{b} + 2\alpha\,\frac{b_{xy}}{b} - (\beta-\gamma)\,\frac{c_{xx}}{c} - 2\,(\alpha-\gamma)\,\frac{c_{xy}}{c} \\
& + \alpha^2\,\delta + 2\,\alpha\,\beta\,\gamma - 2\,\alpha\,\gamma\,\delta - \beta\,\gamma^2, \\
\mu \ :=\ & -2\,\delta\,\frac{a_{xy}}{a} - \gamma\,\frac{a_{yy}}{a} + 2\,\beta\,\frac{b_{xy}}{b} + \alpha\,\frac{b_{yy}}{b} - 2\,(\beta-\delta)\,\frac{c_{xy}}{c} - (\alpha-\gamma)\,\frac{c_{yy}}{c} \\
& + 2\,\alpha\,\beta\,\delta - \alpha\,\delta^2 + \beta^2\,\gamma - 2\,\beta\,\gamma\,\delta, \\
\nu \ :=\ & -\delta\,\frac{a_{yy}}{a} + \beta\,\frac{b_{yy}}{b} - (\beta-\delta)\,\frac{c_{yy}}{c} \\
& + \beta^2\,\delta - \beta\,\delta^2.
\end{aligned}
$$

Proof. This is verified by a direct computation. □

In the sequel, these (a bit complicated) coefficients κ, λ, μ, ν will play a crucial role. Before really treating jets of order 2, it is advisable to understand first jets of order 1.

2. Jet of Order 1

In the complement $\mathbb{C}^2\backslash\{D=0\}$, a first order jet differential is a linear combination of the generators shown above:

$$
\begin{aligned}
\mathsf{Jet}_{abc} \ :=\ & \sum_{i+j=m} R_{i,j}\,\Big(\big(\log\frac{a}{c}\big)'\Big)^i\,\Big(\big(\log\frac{b}{c}\big)'\Big)^j \\
=\ & \sum_{i+j=m} R_{i,j}\,\overline{x}'^i\,\overline{y}'^j.
\end{aligned}
$$

Here, the $R_{i,j}$ should be holomorphic (no singularity) outside $\{D=0\}$. The GAGA principle then insures that *each $R_{i,j}$ is a rational expressions with denominator a certain integer power of D.* Our preliminary objective is to determine these integers.

To this aim, we should perform the transfer to the standard affine first jet coordinates:

$$
\sum_{i+j=m} R_{i,j}\,\big(\alpha\,x'+\beta\,y'\big)^i\,\big(\gamma\,x'+\delta\,y'\big)^j \ =\ \sum_{i+j=m} (\mathbf{?})\,x'^i\,y'^j,
$$

for instance when $m=1$:

$$
\begin{aligned}
R_{1,0}\,\big(\alpha\,x'+\beta\,y'\big) + R_{0,1}\,\big(\gamma\,x'+\delta\,y'\big) \ =\ & x'\,\big[\alpha\,R_{1,0}+\gamma\,R_{0,1}\big] + y'\,\big[\beta\,R_{1,0}+\delta\,R_{0,1}\big] \\
=:\ & x'\,\overline{R}_{1,0} + y'\,\overline{R}_{0,1},
\end{aligned}
$$

so that:

$$
\begin{pmatrix} \alpha & \gamma \\ \beta & \delta \end{pmatrix}\,\begin{pmatrix} R_{1,0} \\ R_{0,1} \end{pmatrix} \ =\ \begin{pmatrix} \overline{R}_{1,0} \\ \overline{R}_{0,1}. \end{pmatrix}
$$

For general $m\geqslant 1$, let us give the names $\overline{R}_{i,j}$ to the transferred coefficients, so that:

$$
\sum_{i+j=m} R_{i,j}\,\overline{x}'^i\,\overline{y}'^j \ =\ \sum_{i+j=m} \overline{R}_{i,j}\,x'^i\,y'^j.
$$

When $m = 2$, a similar computation gives:

$$\begin{pmatrix} \alpha^2 & \alpha\gamma & \gamma^2 \\ 2\alpha\beta & \alpha\delta + \beta\,\gamma & 2\gamma\delta \\ \beta^2 & \beta\delta & \delta^2 \end{pmatrix} \begin{pmatrix} R_{2,0} \\ R_{1,1} \\ R_{0,2} \end{pmatrix} = \begin{pmatrix} \overline{R}_{2,0} \\ \overline{R}_{1,1} \\ \overline{R}_{0,2} \end{pmatrix},$$

while for $m = 3$:

$$\begin{pmatrix} \alpha^3 & \alpha^2\gamma & \alpha\gamma^2 & \gamma^3 \\ 2\alpha^2\beta & \alpha^2\delta + 2\alpha\beta\gamma & 2\alpha\gamma\delta + \beta\gamma^2 & 3\gamma^2\delta \\ 3\alpha\beta^2 & 2\alpha\beta\gamma + \beta^2\gamma & \alpha\delta^2 + 2\beta\gamma\delta & 3\gamma\delta^2 \\ \beta^3 & \beta^2\delta & \beta\delta^2 & \delta^3 \end{pmatrix} \begin{pmatrix} R_{3,0} \\ R_{2,1} \\ R_{1,2} \\ R_{0,3} \end{pmatrix} = \begin{pmatrix} \overline{R}_{3,0} \\ \overline{R}_{2,1} \\ \overline{R}_{1,2} \\ \overline{R}_{0,3} \end{pmatrix}.$$

These are (known) matrices of linear representations of $\mathsf{GL}(2, \mathbb{C})$ on binary forms of degrees 1, 2, 3.

Remember that α, β, γ, δ are holomorphic outside the divisor:

$$\mathscr{D} \cap \{T \neq 0\} = \{a = 0\} \cup \{b = 0\} \cup \{c = 0\}$$

because only a, b, c appear in their denorminators.

Hypothesis 2.1. *The main assumption is that, in the standard affine coordinates* $(x, y) \in \mathbb{C}^2$*, the jet differential be* holomorphic all over $\mathbb{C}^2 \backslash \{abc = 0\}$.

Therefore, all coefficients:

$$\overline{R}_{i,j}$$

should be holomorphic, and hence by the GAGA principle rational with only a, b, c allowed in their denominators.

Coming back to $m = 1$, the inversion:

$$\begin{aligned} \begin{pmatrix} R_{1,0} \\ R_{0,1} \end{pmatrix} &= \frac{1}{\alpha\delta - \beta\gamma} \begin{pmatrix} \delta & -\gamma \\ -\beta & \alpha \end{pmatrix} \begin{pmatrix} \overline{R}_{1,0} \\ \overline{R}_{0,1} \end{pmatrix} \\ &= \frac{1}{D^1} \begin{pmatrix} \delta & -\gamma \\ -\beta & \alpha \end{pmatrix} \begin{pmatrix} \overline{R}_{1,0} \\ \overline{R}_{0,1} \end{pmatrix} \end{aligned}$$

shows that the maximal allowed power of D in $R_{1,0}$, $R_{0,1}$ is equal to 1.

For $m = 2$ and for $m = 3$, a direct inversion of the matrices written above would show, analogously, that the maximal allowed power of D in the $\{R_{i,j}\}_{i+j=2}$ and in the $\{R_{i,j}\}_{i+j=3}$ is equal to 2 and to 3. Yet, to treat any $m \geqslant 1$ (still only for jets of order 1 in this section), one should invert a certain $(m + 1) \times (m + 1)$ matrix, a task which could be delicate to perform.

For the time being, let us at least write what a direct inversion gives when $m = 2$:

$$\begin{pmatrix} R_{2,0} \\ R_{1,1} \\ R_{0,2} \end{pmatrix} = \underbrace{\frac{1}{(\alpha\delta - \beta\gamma)^2}}_{= \frac{1}{D^2}} \begin{pmatrix} \delta^2 & -\gamma\delta & \gamma^2 \\ -2\beta\delta & \alpha\delta + \beta\gamma & -2\alpha\gamma \\ \beta^2 & -\alpha\beta & \alpha^2 \end{pmatrix} \begin{pmatrix} \overline{R}_{2,0} \\ \overline{R}_{1,1} \\ \overline{R}_{0,2} \end{pmatrix}$$

and when $m = 3$:

$$\begin{pmatrix} R_{3,0} \\ R_{2,1} \\ R_{1,2} \\ R_{0,3} \end{pmatrix} = \underbrace{\frac{1}{(\alpha\delta-\beta\gamma)^3}}_{=\frac{1}{D^3}} \begin{pmatrix} \delta^3 & -\gamma\delta^2 & \gamma^2\delta & -\gamma^3 \\ -3\beta\delta^2 & \alpha\delta^2+2\beta\gamma\delta & -2\alpha\gamma\delta-\beta\gamma^2 & 3\alpha\gamma^2 \\ 3\beta^2\delta & -2\alpha\beta\delta-\beta^2\gamma & \alpha^2\delta+2\alpha\beta\gamma & -3\alpha^2\gamma \\ -\beta^3 & \alpha\beta^2 & -\alpha^2\beta & \alpha^3 \end{pmatrix} \begin{pmatrix} \overline{R}_{3,0} \\ \overline{R}_{2,1} \\ \overline{R}_{1,2} \\ \overline{R}_{0,3} \end{pmatrix}.$$

Next, let us attempt to write down the $(m+1)\times(m+1)$ matrix, depending on α, β, γ, δ, which shows how the $\big\{\overline{R}_{i,j}\big\}_{i+j=m}$ express (linearly) in terms of the $\big\{R_{k,l}\big\}_{k+l=m}$. This is done by examining the fundamental *transfer equation* and by expanding:

$$\begin{aligned}
\sum_{i+j=m} \overline{R}_{i,j}\, x'^i\, y'^j &= \sum_{i+j=m} R_{i,j}\, \overline{x}'^i\, \overline{y}'^j \\
&= \sum_{i+j=m} R_{i,j}\, \big(\alpha\, x' + \beta\, y'\big)^i \big(\gamma\, x' + \delta\, y'\big)^j \\
&= \sum_{i+j=m} R_{i,j} \sum_{i_1+i_2=i} \alpha^{i_1}\, x'^{i_1}\, \beta^{i_2}\, y'^{i_2} \tbinom{i_1+i_2}{i_1} \sum_{j_1+j_2=j} \gamma^{j_1}\, x'^{j_1}\, \delta^{j_2}\, y'^{j_2} \tbinom{j_1+j_2}{j_1} \\
&= \sum_{i_1+i_2+j_1+j_2=m} x'^{i_1+j_1}\, y'^{i_2+j_2} \Big[\tbinom{i_1+i_2}{i_1}\tbinom{j_1+j_2}{j_1}\, \alpha^{i_1}\, \beta^{i_2}\, \gamma^{j_1}\, \delta^{j_2}\, R_{i_1+i_2,j_1+j_2}\Big] \\
&= \sum_{i+j=m} x'^i\, y'^j \bigg(\sum_{i_1+j_1=i}\ \sum_{i_2+j_2=j} \tbinom{i_1+i_2}{i_1}\tbinom{j_1+j_2}{j_1}\, \alpha^{i_1}\, \beta^{i_2}\, \gamma^{j_1}\, \delta^{j_2}\, R_{i_1+i_2,j_1+j_2}\bigg).
\end{aligned}$$

All the obtained expressions within the large parentheses are linear with respect to $R_{m,0}, \dots, R_{0,m}$, and we decide to summarize the result as:

$$\sum_{i+j=m} x'^i\, y'^j\, \overline{R}_{i,j} = \sum_{i+j=m} x'^i\, y'^j \bigg(\sum_{k+l=m} \Lambda_{i,j,k,l}(\alpha,\gamma,\beta,\delta)\, R_{k,l}\bigg),$$

without trying to formulate precisely how these $\Lambda_{i,j,k,l}$ come from $\sum_{i_1+j_1=i}\sum_{i_2+j_2=j}$, which would be useless. Therefore by identification:

$$\overline{R}_{i,j} = \sum_{k+l=m} \Lambda_{i,j,k,l}(\alpha,\gamma,\beta,\delta)\, R_{k,l} \qquad {\scriptstyle (i+j=m)}.$$

Observation 2.2. *Then with the* same *functions $\Lambda_{i,j,k,l}$ of 4 variables, the inverse formulas are:*

$$\sum_{k+l=m} \frac{\Lambda_{i,j,k,l}\big(\delta,-\gamma,-\beta,\alpha\big)}{(\alpha\delta-\beta\gamma)^m}\, \overline{R}_{k,l} = R_{i,j} \qquad {\scriptstyle (i+j=m)}.$$

Proof. Indeed, starting from:

$$\begin{aligned} \overline{x}' &= \alpha\, x' + \beta\, y', \\ \overline{y}' &= \gamma\, x' + \delta\, y' \end{aligned} \qquad \Longleftrightarrow \qquad \begin{aligned} \tfrac{\delta}{\alpha\delta-\beta\gamma}\, \overline{x}' - \tfrac{\beta}{\alpha\delta-\beta\gamma}\, \overline{y}' &= x', \\ -\tfrac{\gamma}{\alpha\delta-\beta\gamma}\, \overline{x}' + \tfrac{\alpha}{\alpha\delta-\beta\gamma}\, \overline{y}' &= y', \end{aligned}$$

the inverse formulas are found simply by expanding in a totally parallel manner:

$$\sum_{i+j=m} \overline{R}_{i,j} \Big(\tfrac{\delta}{\alpha\delta-\beta\gamma}\, \overline{x}' - \tfrac{\beta}{\alpha\delta-\beta\gamma}\, \overline{y}'\Big)^i \Big(-\tfrac{\gamma}{\alpha\delta-\beta\gamma}\, \overline{x}' + \tfrac{\alpha}{\alpha\delta-\beta\gamma}\, \overline{y}'\Big)^j = \sum_{i+j=m} R_{i,j}\, \overline{x}'^i\, \overline{y}'^j. \qquad \square$$

Thus, these linear transfer formulas show that the allowed maximal power of $D = \alpha\delta - \beta\gamma$ in the denominators of the $R_{i,j}$ is equal to m. In conclusion:

Proposition 2.3. *In the affine chart $(x,y) \in \mathbb{C}^2 \subset \mathbb{P}^2$ minus the Jacobian zero-locus:*

$$\big\{D=0\big\} \;=\; \big\{J=0\big\} \cap \{T \neq 1\},$$

the coefficients $R_{i,j}$ of any 1st order logarithmic jet differential defined and holomorphic outside $\{abc=0\}$ with total number of primes equal to $m \geqslant 1$:

$$\mathsf{Jet}_{abc} \;=\; \sum_{i+j=m} R_{i,j}\,\Big(\big(\log\frac{a}{c}\big)'\Big)^i\,\Big(\big(\log\frac{b}{c}\big)'\Big)^j,$$

and with a, b, c allowed in denominator places, are rational expressions incorporating as maximal singularity:

$$\frac{1}{D^m} \;=\; \frac{1}{(\alpha\delta-\beta\gamma)^m}. \qquad \square$$

For instance when $m=4$:

$$\begin{pmatrix} R_{4,0} \\ R_{3,1} \\ R_{2,2} \\ R_{1,3} \\ R_{0,4} \end{pmatrix} \;=\; \frac{1}{D^4} \begin{pmatrix} \delta^4 & -\gamma\delta^3 & \gamma^2\delta^2 & -\gamma^3\delta & \gamma^4 \\ -4\beta\delta^3 & \alpha\delta^3+3\beta\gamma\delta^2 & -2\alpha\gamma\delta^2-2\beta\gamma^2\delta & 3\alpha\gamma^2\delta+\beta\gamma^3 & -4\alpha\gamma^3 \\ 6\beta^2\delta^2 & -3\alpha\beta\delta^2-3\beta^2\gamma\delta & \alpha^2\delta^2+4\alpha\beta\gamma\delta+\beta^2\gamma^2 & -3\alpha^2\gamma\delta-3\alpha\beta\gamma^2 & 6\alpha^2\gamma^2 \\ -4\beta^3\delta & 3\alpha\beta^2\delta+\beta^3\gamma & -2\alpha^2\beta\delta-2\alpha\beta^2\gamma & \alpha^3\delta+3\alpha^2\beta\gamma & -4\alpha^3\gamma \\ \beta^4 & -\alpha\beta^3 & \alpha^2\beta^2 & -\alpha^3\beta & \alpha^4 \end{pmatrix} \begin{pmatrix} \overline{R}_{4,0} \\ \overline{R}_{3,1} \\ \overline{R}_{2,2} \\ \overline{R}_{1,3} \\ \overline{R}_{0,4} \end{pmatrix},$$

the polynomials a, b, c being present in denominator places inside:

$$\alpha \;:=\; \frac{a_x\,c-a\,c_x}{a\,c}, \qquad \beta \;:=\; \frac{a_y\,c-a\,c_y}{a\,c},$$
$$\beta \;:=\; \frac{b_x\,c-b\,c_x}{b\,c}, \qquad \delta \;:=\; \frac{b_y\,c-b\,c_y}{b\,c}.$$

For later purposes, it will be convenient to write:

$$\overline{R}_{i,j} \;=\; \sum_{k+l=m} \Pi_{i,j,k,l}\,R_{k,l} \qquad {\scriptstyle (i+j=m)},$$

where:

$$\Pi_{i,j,k,l} \;:=\; \Lambda_{i,j,k,l}\big(\alpha,\gamma,\beta,\delta\big),$$

and also, we will write the inverse formulas as:

$$\sum_{k+l=m} \frac{1}{D^m}\,\overline{\Pi}_{i,j,k,l}\,\overline{R}_{k,l} \;=\; R_{i,j} \qquad {\scriptstyle (i+j=m)},$$

with by what precedes:

$$\overline{\Pi}_{i,j,k,l} \;:=\; \Lambda_{i,j,k,l}\big(\delta,-\gamma,-\beta,\alpha\big).$$

More concise formulas are welcome. Since $i+j=m$ and $k+l=m$ anyway, we can erase i and we can erase k. Yet, we will indicate the considered m by putting it in upper-case place, so that we will write:

$$\overline{R}_j^m \;=\; \sum_{0\leqslant l\leqslant m} \Pi_{j,l}^m\,R_l^m \qquad {\scriptstyle (0\leqslant j\leqslant m)},$$

$$\sum_{0\leqslant l\leqslant m} \frac{1}{D^m}\,\overline{\Pi}_{j,l}^m\,\overline{R}_l^m \;=\; R_j^m \qquad {\scriptstyle (0\leqslant j\leqslant m)}.$$

3. Weighted Degree $m = 3$ and Jets of Order 2

Assuming $m = 3$ and working now with jets of order 2, let us start from:

$$
\begin{aligned}
\mathsf{Jet}_{abc} \;&:=\; \sum_{i+j=3} R_{i,j}\,\Big(\big(\log\tfrac{a}{c}\big)'\Big)^i\,\Big(\big(\log\tfrac{b}{c}\big)'\Big)^j + S_{0,0}\,\begin{vmatrix} \big(\log\frac{a}{c}\big)' & \big(\log\frac{b}{c}\big)' \\ \big(\log\frac{a}{c}\big)'' & \big(\log\frac{b}{c}\big)'' \end{vmatrix} \\
&=\; \sum_{i+j=3} R_{i,j}\,\overline{x}'^i\,\overline{y}'^j + S_{0,0}\,\overline{W}''',
\end{aligned}
$$

and let us perform the *transfer* to the standard affine jet coordinates (x, y, x', y', x'', y''), which amounts to expanding and reorganizing:

$$
\begin{aligned}
\sum_{i+j=3} R_{i,j}\,\overline{x}'^i\,\overline{y}'^j + S_{0,0}\,\overline{W}''' \;=\; &\sum_{i+j=3} R_{i,j}\,\big(\alpha\,x' + \beta\,y'\big)^i\,\big(\gamma\,x' + \delta\,y'\big)^j \\
&+ S_{0,0}\,\Big((\alpha\delta - \beta\gamma)\,W''' + \kappa\,{x'}^3 + \lambda\,{x'}^2 y' + \mu\,x'{y'}^2 + \nu\,{y'}^3\Big),
\end{aligned}
$$

where we have abbreviated the standard Wronskian as:

$$
W''' \;:=\; \begin{vmatrix} x' & y' \\ x'' & y'' \end{vmatrix} \;=\; x'\,y'' - x''\,y'.
$$

Using our formalism, we receive the equation:

$$
\begin{aligned}
&\sum_{i+j=m} R_{i,j}\,\big(\alpha\,x' + \beta\,y'\big)^i\,\big(\gamma\,x' + \delta\,y'\big)^j \\
&\quad + S_{0,0}\,\Big((\alpha\delta - \beta\gamma)\,W''' + \kappa\,{x'}^3 + \lambda\,{x'}^2 y' + \mu\,x'{y'}^2 + \nu\,{y'}^3\Big) \;=\; \sum_{i+j=3} \overline{R}_{i,j}\,x'^i\,y'^j + \overline{S}_{0,0}\,W''',
\end{aligned}
$$

whence firstly by identification of the coefficients of W''':

$$
S_{0,0}\,D \;=\; \overline{S}_{0,0},
$$

and secondly by identification of the powers of x', y' (in matrix notation):

$$
\begin{pmatrix}
\alpha^3 & \alpha^2\gamma & \alpha\gamma^2 & \gamma^3 \\
2\alpha^2\beta & \alpha^2\delta + 2\alpha\beta\gamma & 2\alpha\gamma\delta + \beta\gamma^2 & 3\gamma^2\delta \\
3\alpha\beta^2 & 2\alpha\beta\gamma + \beta^2\gamma & \alpha\delta^2 + 2\beta\gamma\delta & 3\gamma\delta^2 \\
\beta^3 & \beta^2\delta & \beta\delta^2 & \delta^3
\end{pmatrix}
\begin{pmatrix} R_{3,0} \\ R_{2,1} \\ R_{1,2} \\ R_{0,3} \end{pmatrix}
+
\begin{pmatrix} \kappa\,S_{0,0} \\ \lambda\,S_{0,0} \\ \mu\,S_{0,0} \\ \nu\,S_{0,0} \end{pmatrix}
=
\begin{pmatrix} \overline{R}_{3,0} \\ \overline{R}_{2,1} \\ \overline{R}_{1,2} \\ \overline{R}_{0,3}. \end{pmatrix}
$$

Here, we should solve the $R_{i,j}$ in terms of the $\overline{R}_{k,l}$ and of $\overline{S}_{0,0}$. At first, we substitute $S_{0,0}$ in terms of $\overline{S}_{0,0}$ and we reorganize this matrix equation as:

$$
\begin{pmatrix}
\alpha^3 & \alpha^2\gamma & \alpha\gamma^2 & \gamma^3 \\
2\alpha^2\beta & \alpha^2\delta + 2\alpha\beta\gamma & 2\alpha\gamma\delta + \beta\gamma^2 & 3\gamma^2\delta \\
3\alpha\beta^2 & 2\alpha\beta\gamma + \beta^2\gamma & \alpha\delta^2 + 2\beta\gamma\delta & 3\gamma\delta^2 \\
\beta^3 & \beta^2\delta & \beta\delta^2 & \delta^3
\end{pmatrix}
\begin{pmatrix} R_{3,0} \\ R_{2,1} \\ R_{1,2} \\ R_{0,3} \end{pmatrix}
=
\begin{pmatrix} \overline{R}_{3,0} \\ \overline{R}_{2,1} \\ \overline{R}_{1,2} \\ \overline{R}_{0,3} \end{pmatrix}
-
\begin{pmatrix} \kappa\,\frac{1}{\alpha\delta-\beta\gamma}\,\overline{S}_{0,0} \\ \lambda\,\frac{1}{\alpha\delta-\beta\gamma}\,\overline{S}_{0,0} \\ \mu\,\frac{1}{\alpha\delta-\beta\gamma}\,\overline{S}_{0,0} \\ \nu\,\frac{1}{\alpha\delta-\beta\gamma}\,\overline{S}_{0,0} \end{pmatrix}.
$$

Next, since we know well the inverse of this 3×3 matrix, we may solve:

$$\begin{pmatrix} \overline{R}_{3,0} \\ \overline{R}_{2,1} \\ \overline{R}_{1,2} \\ \overline{R}_{0,3} \end{pmatrix} = \underbrace{\frac{1}{(\alpha\delta-\beta\gamma)^3}}_{=\frac{1}{D^3}} \begin{pmatrix} \delta^3 & -\gamma\delta^2 & \gamma^2\delta & -\gamma^3 \\ -3\beta\delta^2 & \alpha\delta^2+2\beta\gamma\delta & -2\alpha\gamma\delta-\beta\gamma^2 & 3\alpha\gamma^2 \\ 3\beta^2\delta & -2\alpha\beta\delta-\beta^2\gamma & \alpha^2\delta+2\alpha\beta\gamma & -3\alpha^2\gamma \\ -\beta^3 & \alpha\beta^2 & -\alpha^2\beta & \alpha^3 \end{pmatrix} \begin{pmatrix} \overline{R}_{3,0} \\ \overline{R}_{2,1} \\ \overline{R}_{1,2} \\ \overline{R}_{0,3} \end{pmatrix}$$

$$- \frac{1}{(\alpha\delta-\beta\gamma)^3} \frac{\overline{S}_{0,0}}{(\alpha\delta-\beta\gamma)^1} \begin{pmatrix} \delta^3 & -\gamma\delta^2 & \gamma^2\delta & -\gamma^3 \\ -3\beta\delta^2 & \alpha\delta^2+2\beta\gamma\delta & -2\alpha\gamma\delta-\beta\gamma^2 & 3\alpha\gamma^2 \\ 3\beta^2\delta & -2\alpha\beta\delta-\beta^2\gamma & \alpha^2\delta+2\alpha\beta\gamma & -3\alpha^2\gamma \\ -\beta^3 & \alpha\beta^2 & -\alpha^2\beta & \alpha^3 \end{pmatrix} \begin{pmatrix} \kappa \\ \lambda \\ \mu \\ \nu \end{pmatrix}.$$

Looking at this formula, it seems that the maximal appearing power of the reciprocal affine Jacobian determinant $\frac{1}{D} = \frac{1}{\alpha\delta-\beta\gamma}$ is equal to $3+1=4$. But let us examine more closely:

$$\begin{pmatrix} \delta^3 & -\gamma\delta^2 & \gamma^2\delta & -\gamma^3 \\ -3\beta\delta^2 & \alpha\delta^2+2\beta\gamma\delta & -2\alpha\gamma\delta-\beta\gamma^2 & 3\alpha\gamma^2 \\ 3\beta^2\delta & -2\alpha\beta\delta-\beta^2\gamma & \alpha^2\delta+2\alpha\beta\gamma & -3\alpha^2\gamma \\ -\beta^3 & \alpha\beta^2 & -\alpha^2\beta & \alpha^3 \end{pmatrix} \begin{pmatrix} \kappa \\ \lambda \\ \mu \\ \nu \end{pmatrix} =: \begin{pmatrix} \tau_{3,0} \\ \tau_{2,1} \\ \tau_{1,2} \\ \tau_{0,3} \end{pmatrix}.$$

Miracle Factorizations 3.1. *The 4 expressions:*

$$\begin{aligned}
\tau_{3,0} &:= \delta^3\,\kappa - \gamma\,\delta^2\,\lambda + \gamma^2\,\delta\,\mu - \gamma^3\,\nu \\
&= \big(\alpha\,\delta - \beta\,\gamma\big)\,\big(\cdots\big), \\
\tau_{2,1} &:= -\,3\,\beta\,\delta^2\,\kappa + \big(\alpha\,\delta^2 + 2\,\beta\,\gamma\,\delta\big)\,\lambda + \big(-\,2\,\alpha\,\gamma\,\delta - \beta\,\gamma^2\big)\,\mu + 3\,\alpha\,\gamma^2\,\nu \\
&= \big(\alpha\,\delta - \beta\,\gamma\big)\,\big(\cdots\big), \\
\tau_{1,2} &:= 3\,\beta^2\,\delta\,\kappa + \big(-2\,\alpha\,\beta\,\delta - \beta^2\,\gamma\big)\,\lambda + \big(\alpha^2\,\delta + 2\,\alpha\,\beta\,\gamma\big)\,\mu - 3\,\alpha^2\,\gamma\,\nu \\
&= \big(\alpha\,\delta - \beta\,\gamma\big)\,\big(\cdots\big), \\
\tau_{0,3} &:= -\,\beta^3\,\kappa + \alpha\,\beta^2\,\lambda - \alpha^2\,\beta\,\mu + \alpha^3\,\nu \\
&= \big(\alpha\,\delta - \beta\,\gamma\big)\,\big(\cdots\big),
\end{aligned}$$

are all multiples of $\alpha\delta - \beta\gamma = D$.

Proof. Indeed, by a direct computation:

$$\tau_{3,0} = \big(\alpha\,\delta - \beta\,\gamma\big)\left(\delta^2\,\frac{c_{xx}}{c} - 2\,\gamma\,\delta\,\frac{c_{xy}}{c} + \gamma^2\,\frac{c_{yy}}{c} - \delta^2\,\frac{b_{xx}}{b} + 2\,\gamma\,\delta\,\frac{b_{xy}}{b} - \gamma^2\,\frac{b_{yy}}{b}\right),$$

$$\begin{aligned}
\tau_{2,1} = \big(\alpha\,\delta - \beta\,\gamma\big)\bigg(&-\,\alpha^2\,\delta^2 + 2\,\alpha\,\beta\,\gamma\,\delta - \beta^2\,\gamma^2 + 2\,\alpha\,\delta\,\frac{c_{xy}}{c} - 2\,\alpha\,\gamma\,\frac{c_{yy}}{c} - 2\,\alpha\,\delta\,\frac{b_{xy}}{b} \\
&+ 2\,\alpha\,\gamma\,\frac{b_{yy}}{b} - 2\,\beta\,\delta\,\frac{c_{xx}}{c} + 2\,\beta\,\gamma\,\frac{c_{xy}}{c} - \delta^2\,\frac{c_{xx}}{c} + 2\,\gamma\,\delta\,\frac{c_{xy}}{c} - \gamma^2\,\frac{c_{yy}}{c} \\
&+ 2\,\beta\,\delta\,\frac{b_{xx}}{b} - 2\,\beta\,\gamma\,\frac{b_{xy}}{b} + \delta^2\,\frac{a_{xx}}{a} - 2\,\gamma\,\delta\,\frac{a_{xy}}{a} + \gamma^2\,\frac{a_{yy}}{a}\bigg),
\end{aligned}$$

$$\begin{aligned}
\tau_{1,2} = \big(\alpha\,\delta - \beta\,\gamma\big)\bigg(&\alpha^2\,\delta^2 - 2\,\alpha\,\beta\,\gamma\,\delta + \beta^2\,\gamma^2 + \alpha^2\,\frac{c_{yy}}{c} - \alpha^2\,\frac{b_{yy}}{b} - 2\,\alpha\,\beta\,\frac{c_{xy}}{c} \\
&-\,2\,\alpha\,\delta\,\frac{c_{xy}}{c} + 2\,\alpha\,\gamma\,\frac{c_{yy}}{c} + 2\,\alpha\,\beta\,\frac{b_{xy}}{b} + \beta^2\,\frac{c_{xx}}{c} + 2\,\beta\,\delta\,\frac{c_{xx}}{c} - 2\,\beta\,\gamma\,\frac{c_{xy}}{c} \\
&-\,\beta^2\,\frac{b_{xx}}{b} + 2\,\alpha\,\delta\,\frac{a_{xy}}{a} - 2\,\alpha\,\gamma\,\frac{a_{yy}}{a} - 2\,\beta\,\delta\,\frac{a_{xx}}{a} + 2\,\beta\,\gamma\,\frac{a_{xy}}{a}\bigg),
\end{aligned}$$

$$\tau_{0,3} \,=\, \big(\alpha\,\delta-\beta\,\gamma\big)\bigg(-\,\alpha^2\,\frac{c_{yy}}{c}+2\,\alpha\,\beta\,\frac{c_{xy}}{c}-\beta^2\,\frac{c_{xx}}{c}+\alpha^2\,\frac{a_{yy}}{a}-2\,\alpha\,\beta\,\frac{a_{xy}}{a}+\beta^2\,\frac{a_{xx}}{a}\bigg),$$

we find the four missing $\big(\cdots\big)$. □

Therefore:

$$\begin{pmatrix}\tau_{3,0}\\ \tau_{2,1}\\ \tau_{1,2}\\ \tau_{0,3}\end{pmatrix} \;=:\; \big(\alpha\,\delta-\beta\,\gamma\big)\,\begin{pmatrix}\theta_{3,0}\\ \theta_{2,1}\\ \theta_{1,2}\\ \theta_{0,3}\end{pmatrix}$$

and finally:

$$\begin{pmatrix}\overline{R}_{3,0}\\ \overline{R}_{2,1}\\ \overline{R}_{1,2}\\ \overline{R}_{0,3}\end{pmatrix} = \frac{1}{(\alpha\delta-\beta\gamma)^3}\begin{pmatrix}\delta^3 & -\gamma\delta^2 & \gamma^2\delta & -\gamma^3\\ -3\beta\delta^2 & \alpha\delta^2+2\beta\gamma\delta & -2\alpha\gamma\delta-\beta\gamma^2 & 3\alpha\gamma^2\\ 3\beta^2\delta & -2\alpha\beta\delta-\beta^2\gamma & \alpha^2\delta+2\alpha\beta\gamma & -3\alpha^2\gamma\\ -\beta^3 & \alpha\beta^2 & -\alpha^2\beta & \alpha^3\end{pmatrix}\begin{pmatrix}\overline{R}_{3,0}\\ \overline{R}_{2,1}\\ \overline{R}_{1,2}\\ \overline{R}_{0,3}\end{pmatrix}$$
$$-\,\frac{1}{(\alpha\delta-\beta\gamma)^3}\,\frac{\overline{S}_{0,0}}{(\alpha\delta-\beta\gamma)^1}\,\big(\alpha\,\delta-\beta\,\gamma\big)^1\begin{pmatrix}\theta_{3,0}\\ \theta_{2,1}\\ \theta_{1,2}\\ \theta_{0,3}\end{pmatrix}.$$

Proposition 3.2. *In the affine chart $(x,y)\in\mathbb{C}^2\subset\mathbb{P}^2$ minus the Jacobian zero-locus:*

$$\big\{D=0\big\} \,=\, \big\{J=0\big\}\cap\{T\neq 1\},$$

the coefficients $\big\{R_{i,j}\big\}_{i+j=3}$ and $S_{0,0}$ of any 2[nd] order logarithmic jet differential defined and holomorphic outside $\{abc=0\}$ with total number of primes equal to $m=3$:

$$\mathsf{Jet}_{abc} \,=\, \sum_{i+j=3} R_{i,j}\,\Big(\big(\mathsf{log}\,\frac{a}{c}\big)'\Big)^i\,\Big(\big(\mathsf{log}\,\frac{b}{c}\big)'\Big)^j + S_{0,0}\,\begin{vmatrix}\big(\mathsf{log}\,\frac{a}{c}\big)' & \big(\mathsf{log}\,\frac{b}{c}\big)'\\ \big(\mathsf{log}\,\frac{a}{c}\big)'' & \big(\mathsf{log}\,\frac{b}{c}\big)''\end{vmatrix}$$

and with a, b, c allowed in denominator places, are rational expressions incorporating as maximal singularity:

$$\begin{aligned}\frac{1}{D^3} &\,=\, \frac{1}{(\alpha\delta-\beta\gamma)^3} \qquad \textit{in}\;\; R_{i,j},\\ \frac{1}{D^1} &\,=\, \frac{1}{(\alpha\delta-\beta\gamma)^1} \qquad \textit{in}\;\; S_{0,0},.\end{aligned}$$

□

4. Back to Homogeneous Coordinates for Three Conics

Therefore, this Proposition 3.2 shows that a general logarithmic jet differential of weighted order $m=3$ must be the following form in the original homogeneous coordinates:

$$\mathsf{Jet}_{ABC} \,=\, \sum_{i+j=3}\frac{R_{i,j}(T,X,Y)}{\big(J(T,X,Y)\big)^3}\,\Big(\big(\mathsf{log}\,\frac{A}{C}\big)'\Big)^i\,\Big(\big(\mathsf{log}\,\frac{B}{C}\big)'\Big)^j + \frac{S_{0,0}(T,X,Y)}{\big(J(T,X,Y)\big)^1}\,\begin{vmatrix}\big(\mathsf{log}\,\frac{A}{C}\big)' & \big(\mathsf{log}\,\frac{B}{C}\big)'\\ \big(\mathsf{log}\,\frac{A}{C}\big)'' & \big(\mathsf{log}\,\frac{B}{C}\big)''\end{vmatrix},$$

with certain homogeneous *polynomials:*

$$R_{i,j}\,\in\,\mathbb{C}[T,X,Y], \qquad\qquad S_{0,0}\,\in\,\mathbb{C}[T,X,Y],$$

the denominators being *controlled* powers J^3 and J^1 of the Jacobian determinant:

$$J(T,X,Y) = \begin{vmatrix} A_T & B_T & C_T \\ A_X & B_X & C_X \\ A_Y & B_Y & C_Y \end{vmatrix};$$

by slight abuse of notation, we employ the same letters $R_{i,j}$ and $S_{0,0}$ as before.

However, there still remains a crucial necessary condition for such a jet differential Jet_{ABC} to really *be* a *holomorphic* logarithmic jet differential, namely to be *holomorphic everywhere outside the 3-curves locus:*

$$\mathscr{C} := \big\{A\,B\,C = 0\big\},$$

the necessary condition that, after passing to any of the 3 possible affine charts $\{T \neq 0\}$ or $\{X \neq 0\}$ or $\{Y \neq 0\}$, the Jacobians completely disappears in denominator places (while A, B, C are allowed in denominator places by definition of logarithmic jets).

For this to be satisfied, the coefficients of the polynomials $R_{i,j}$ and $S_{0,0}$ must satisfy a certain complicated linear system that we will set up and analyze later. We will in fact focus only on the case $2 = d_A = d_B = d_C$ of three projective conics, where:

$$\mathsf{deg}\, J = 3.$$

For now, in the 3-conics case, let us observe that if we assign constant (homogeneous) degrees:

$$d_R = \mathsf{deg}\, R_{i,j}, \qquad\qquad d_S = \mathsf{deg}\, S_{0,0},$$

with in addition:

$$\begin{aligned} d_R - 3\cdot 3 &= d_S - 3\cdot 1 \\ &=: t, \end{aligned}$$

this Jet_{ABC} written above is a *meromorphic* (rational) section of:

$$E_{2,3}T^*_{\mathbb{P}^2}(\log\mathscr{C}) \otimes \mathscr{O}_{\mathbb{P}^2}(t);$$

indeed, in homogeneous coordinates, the *twist* t of a section σ of a vector bundle $E \longrightarrow \mathbb{P}^2$ is determined by just viewing how it transforms as $\lambda^t\,\sigma$ under a dilation $(T,X,Y) \longmapsto \big(\lambda T, \lambda X, \lambda Y\big)$.

Similarly, Proposition 2.3 read in the 3-conics case shows that a general logarithmic jet differential of jet order 1 and of any weighted order m must be of the following form in the original homogeneous coordinates:

$$\mathsf{Jet}_{ABC} = \sum_{i+j=m} \frac{R_{i,j}(T,X,Y)}{\big(J(T,X,Y)\big)^m} \Big(\big(\log\frac{A}{C}\big)'\Big)^i \Big(\big(\log\frac{B}{C}\big)'\Big)^j,$$

with homogeneous polynomials $R_{i,j} \in \mathbb{C}[T,X,Y]$ — not the same coefficients $R_{i,j}$ as before by slight abuse of notation — of constant degrees:

$$d_R = \mathsf{det}\, R_{i,j} \qquad\qquad {\scriptstyle (i+j=m)},$$

so that this Jet_{ABC} is a *meromorphic* (rational) section of:

$$E_{1,m}T^*_{\mathbb{P}^2}(\log\mathscr{C}) \otimes \mathscr{O}_{\mathbb{P}^2}(t),$$

where:

$$t := d_R - 3\cdot m.$$

5. Weighted Degree $m = 4$, Jet Order 2

Next, with $m = 4$, let us compute the transfer:

$$
\begin{aligned}
\mathsf{Jet}_{abc} \;:=\; & \sum_{i+j=4} R_{i,j} \left(\left(\log\tfrac{a}{c}\right)'\right)^i \left(\left(\log\tfrac{b}{c}\right)'\right)^j \\
& \qquad + S_{1,0}\left(\log\tfrac{a}{c}\right)' \begin{vmatrix} \left(\log\frac{a}{c}\right)' & \left(\log\frac{b}{c}\right)' \\ \left(\log\frac{a}{c}\right)'' & \left(\log\frac{b}{c}\right)'' \end{vmatrix} + S_{0,1}\left(\log\tfrac{b}{c}\right)' \begin{vmatrix} \left(\log\frac{a}{c}\right)' & \left(\log\frac{b}{c}\right)' \\ \left(\log\frac{a}{c}\right)'' & \left(\log\frac{b}{c}\right)'' \end{vmatrix} \\
=\; & \sum_{i+j=4} R_{i,j}\, \overline{x}'^i\, \overline{y}'^j + S_{1,0}\, \overline{x}'\, \overline{W}''' + S_{0,1}\, \overline{y}'\, \overline{W}''' \\
=\; & \sum_{i+j=4} R_{i,j} \left(\alpha\, x' + \beta\, y'\right)^i \left(\gamma\, x' + \delta\, y'\right)^j \\
& \qquad + S_{1,0}\left(\alpha\, x' + \beta\, y'\right)\Big((\alpha\delta - \beta\gamma)\, W''' + \kappa\, x'^3 + \lambda\, x'^2 y' + \mu\, x' y'^2 + \nu\, y'^3\Big) \\
& \qquad + S_{0,1}\left(\gamma\, x' + \delta\, y'\right)\Big((\alpha\delta - \beta\gamma)\, W''' + \kappa\, x'^3 + \lambda\, x'^2 y' + \mu\, x' y'^2 + \nu\, y'^3\Big) \\
=:\; & \sum_{i+j=4} \overline{R}_{i,j}\, x'^i\, y'^j + \overline{S}_{1,0}\, x'\, W''' + \overline{S}_{0,1}\, y'\, W'''.
\end{aligned}
$$

By identification of the coefficients of W''', we get:

$$
S_{1,0}\left(\alpha\, x' + \beta\, y'\right)(\alpha\delta - \beta\gamma) + S_{0,1}\left(\gamma\, x' + \delta\, y'\right)(\alpha\delta - \beta\gamma) \;=\; \overline{S}_{1,0}\, x' + \overline{S}_{0,1}\, y',
$$

hence by identification of the coefficient of x' and y':

$$
\begin{aligned}
(\alpha\delta - \beta\gamma)\left(\alpha\, S_{1,0} + \gamma\, S_{0,1}\right) \;&=\; \overline{S}_{1,0} \\
(\alpha\delta - \beta\gamma)\left(\gamma\, S_{1,0} + \delta\, S_{0,1}\right) \;&=\; \overline{S}_{0,1},
\end{aligned}
$$

whence by inversion:

$$
\begin{pmatrix} S_{1,0} \\ S_{0,1} \end{pmatrix} \;=\; \frac{1}{(\alpha\delta - \beta\gamma)^2} \begin{pmatrix} \delta & -\gamma \\ -\beta & \alpha \end{pmatrix} \begin{pmatrix} \overline{S}_{1,0} \\ \overline{S}_{0,1} \end{pmatrix}.
$$

After subtraction, it remains:

$$
\begin{aligned}
& \sum_{i+j=4} R_{i,j} \left(\alpha\, x' + \beta\, y'\right)^i \left(\gamma\, x' + \delta\, y'\right)^j \\
& \quad + \Big[S_{1,0}\left(\alpha\, x' + \beta\, y'\right) + S_{0,1}\left(\gamma\, x' + \delta\, y'\right)\Big]\Big(\kappa\, x'^3 + \lambda\, x'^2 y' + \mu\, x' y'^2 + \nu\, y'^3\Big) \\
& \quad = \sum_{i+j=4} \overline{R}_{i,j}\, x'^i\, y'^j,
\end{aligned}
$$

that is:

$$
\begin{aligned}
& \sum_{i+j=4} R_{i,j} \left(\alpha\, x' + \beta\, y'\right)^i \left(\gamma\, x' + \delta\, y'\right)^j \\
& \qquad = \sum_{i+j=4} \overline{R}_{i,j}\, x'^i\, y'^j - \frac{\overline{S}_{1,0}}{\alpha\delta - \beta\gamma}\Big(\kappa\, x'^4 + \lambda\, x'^3 y' + \mu\, x'^2 y'^2 + \nu\, x' y'^3\Big) \\
& \qquad\qquad\qquad\qquad - \frac{\overline{S}_{0,1}}{\alpha\delta - \beta\gamma}\Big(\kappa\, x'^3 y' + \lambda\, x'^2 y'^2 + \mu\, x' y'^2 + \nu\, y'^4\Big),
\end{aligned}
$$

or equivalently in matrix notation:

$$\begin{pmatrix} \alpha^4 & \alpha^3\gamma & \alpha^2\gamma^2 & \alpha\gamma^3 & \gamma^4 \\ 4\alpha^3\beta & \alpha^3\delta+3\alpha^2\beta\gamma & 2\alpha^2\gamma\delta+2\alpha\beta\gamma^2 & 3\alpha\gamma^2\delta+\beta\gamma^3 & 4\gamma^3\delta \\ 6\alpha^2\beta^2 & 3\alpha^2\beta\delta+3\alpha\beta^2\gamma & \alpha^2\delta^2+4\alpha\beta\gamma\delta & 3\alpha\gamma\delta^2+3\beta\gamma^2\delta & 6\gamma^2\delta^2 \\ 4\alpha\beta^3 & 3\alpha\beta^2\delta+\beta^3\gamma & 2\alpha\beta\delta^2+2\beta^2\gamma\delta & \alpha\delta^3+3\beta\gamma\delta^2 & 4\gamma\delta^3 \\ \beta^4 & \beta^3\delta & \beta^2\delta^2 & \beta\delta^3 & \delta^4 \end{pmatrix} \begin{pmatrix} R_{4,0} \\ R_{3,1} \\ R_{2,1} \\ R_{1,3} \\ R_{0,4} \end{pmatrix} = \begin{pmatrix} \overline{R}_{4,0} \\ \overline{R}_{3,1} \\ \overline{R}_{2,2} \\ \overline{R}_{1,3} \\ \overline{R}_{0,4} \end{pmatrix} - \frac{\overline{S}_{1,0}}{\alpha\delta-\beta\gamma} \begin{pmatrix} \kappa \\ \lambda \\ \mu \\ \nu \\ 0 \end{pmatrix} - \frac{\overline{S}_{0,1}}{\alpha\delta-\beta\gamma} \begin{pmatrix} 0 \\ \kappa \\ \lambda \\ \mu \\ \nu \end{pmatrix}.$$

A simple inversion yields:

$$\begin{aligned} \begin{pmatrix} R_{4,0} \\ R_{3,1} \\ R_{2,2} \\ R_{1,3} \\ R_{0,4} \end{pmatrix} &= \frac{1}{(\alpha\delta-\beta\gamma)^4} \begin{pmatrix} \delta^4 & -\gamma\delta^3 & \gamma^2\delta^2 & -\gamma^3\delta & \gamma^4 \\ -4\beta\delta^3 & \alpha\delta^3+3\beta\gamma\delta^2 & -2\alpha\gamma\delta^2-2\beta\gamma^2\delta & 3\alpha\gamma^2\delta+\beta\gamma^3 & -4\alpha\gamma^3 \\ 6\beta^2\delta^2 & -3\alpha\beta\delta^2-3\beta^2\gamma\delta & \alpha^2\delta^2+4\alpha\beta\gamma\delta+\beta^2\gamma^2 & -3\alpha^2\gamma\delta-3\alpha\beta\gamma^2 & 6\alpha^2\gamma^2 \\ -4\beta^3\delta & 3\alpha\beta^2\delta+\beta^3\gamma & -2\alpha^2\beta\delta-2\alpha\beta^2\gamma & \alpha^3\delta+3\alpha^2\beta\gamma & -4\alpha^3\gamma \\ \beta^4 & -\alpha\beta^3 & \alpha^2\beta^2 & -\alpha^3\beta & \alpha^4 \end{pmatrix} \begin{pmatrix} \overline{R}_{4,0} \\ \overline{R}_{3,1} \\ \overline{R}_{2,2} \\ \overline{R}_{1,3} \\ \overline{R}_{0,4} \end{pmatrix} \\ &- \frac{1}{(\alpha\delta-\beta\gamma)^4} \frac{\overline{S}_{1,0}}{(\alpha\delta-\beta\gamma)^1} \begin{pmatrix} \delta^4 & -\gamma\delta^3 & \gamma^2\delta^2 & -\gamma^3\delta & \gamma^4 \\ -4\beta\delta^3 & \alpha\delta^3+3\beta\gamma\delta^2 & -2\alpha\gamma\delta^2-2\beta\gamma^2\delta & 3\alpha\gamma^2\delta+\beta\gamma^3 & -4\alpha\gamma^3 \\ 6\beta^2\delta^2 & -3\alpha\beta\delta^2-3\beta^2\gamma\delta & \alpha^2\delta^2+4\alpha\beta\gamma\delta+\beta^2\gamma^2 & -3\alpha^2\gamma\delta-3\alpha\beta\gamma^2 & 6\alpha^2\gamma^2 \\ -4\beta^3\delta & 3\alpha\beta^2\delta+\beta^3\gamma & -2\alpha^2\beta\delta-2\alpha\beta^2\gamma & \alpha^3\delta+3\alpha^2\beta\gamma & -4\alpha^3\gamma \\ \beta^4 & -\alpha\beta^3 & \alpha^2\beta^2 & -\alpha^3\beta & \alpha^4 \end{pmatrix} \begin{pmatrix} \kappa \\ \lambda \\ \mu \\ \nu \\ 0 \end{pmatrix} \\ &- \frac{1}{(\alpha\delta-\beta\gamma)^4} \frac{\overline{S}_{0,1}}{(\alpha\delta-\beta\gamma)^1} \begin{pmatrix} \delta^4 & -\gamma\delta^3 & \gamma^2\delta^2 & -\gamma^3\delta & \gamma^4 \\ -4\beta\delta^3 & \alpha\delta^3+3\beta\gamma\delta^2 & -2\alpha\gamma\delta^2-2\beta\gamma^2\delta & 3\alpha\gamma^2\delta+\beta\gamma^3 & -4\alpha\gamma^3 \\ 6\beta^2\delta^2 & -3\alpha\beta\delta^2-3\beta^2\gamma\delta & \alpha^2\delta^2+4\alpha\beta\gamma\delta+\beta^2\gamma^2 & -3\alpha^2\gamma\delta-3\alpha\beta\gamma^2 & 6\alpha^2\gamma^2 \\ -4\beta^3\delta & 3\alpha\beta^2\delta+\beta^3\gamma & -2\alpha^2\beta\delta-2\alpha\beta^2\gamma & \alpha^3\delta+3\alpha^2\beta\gamma & -4\alpha^3\gamma \\ \beta^4 & -\alpha\beta^3 & \alpha^2\beta^2 & -\alpha^3\beta & \alpha^4 \end{pmatrix} \begin{pmatrix} 0 \\ \kappa \\ \lambda \\ \mu \\ \nu \end{pmatrix}. \end{aligned}$$

Again at denominator places, there occurs a D-power discrepancy:

$$4 \,<\, 4+1.$$

But to fix this problem, remind that the 5×5 matrix written above (and copied three times) is:

$$\Big(\overline{\Pi}_{j,l}^{4}\Big)_{0\leqslant j\leqslant 4}^{0\leqslant l\leqslant 5}.$$

Miracle Factorizations 5.1. *All the* $5+5$ *expressions:*

$$\begin{aligned} \tau_{4-j,j}^{1,0} &:= \overline{\Pi}_{j,0}^{4}\,\kappa + \overline{\Pi}_{j,1}^{4}\,\lambda + \overline{\Pi}_{j,2}^{4}\,\mu + \overline{\Pi}_{j,3}^{4}\,\nu &&= \big(\alpha\,\delta-\beta\,\gamma\big)^1\,\theta_{4-j,j}^{1,0} && {\scriptstyle (0\leqslant j\leqslant 4)}, \\ \tau_{4-j,j}^{0,1} &:= \overline{\Pi}_{j,1}^{4}\,\kappa + \overline{\Pi}_{j,2}^{4}\,\lambda + \overline{\Pi}_{j,3}^{4}\,\mu + \overline{\Pi}_{j,4}^{4}\,\nu &&= \big(\alpha\,\delta-\beta\,\gamma\big)^1\,\theta_{4-j,j}^{0,1} && {\scriptstyle (0\leqslant l\leqslant 4)}, \end{aligned}$$

are factorizable by:

$$\big(\alpha\,\delta-\beta\,\gamma\big)^1 \,=\, D^1.$$

We will realize that these $5+5$ further factorizations are *inherited* from the initial 4 Miracle Factorizations 3.1:

$$\tau_{3-j,j}^{0,0} := \overline{\Pi}_{j,0}\,\kappa + \overline{\Pi}_{j,1}\,\lambda + \overline{\Pi}_{j,2}\,\mu + \overline{\Pi}_{j,3} \,=\, \big(\alpha\,\delta-\beta\,\gamma\big)\,\theta_{3-j,j}^{0,0} \qquad {\scriptstyle (0\,\leqslant\, j\,\leqslant\, 3)}.$$

and to this aim, we need some preparation.

Lemma 5.2. *The matrix coefficients* $\Pi_{\bullet,\bullet}^{\bullet}$ *satisfy:*

$$\begin{aligned} \beta\,\Pi_{j-1,l}^{m} + \alpha\,\Pi_{j,l}^{m} &\,=\, \Pi_{j,l}^{m+1} && {\scriptstyle (0\,\leqslant\, j\,\leqslant\, m+1,\ 0\,\leqslant\, l\,\leqslant\, m)}, \\ \delta\,\Pi_{j-1,l}^{m} + \gamma\,\Pi_{j,l}^{m} &\,=\, \Pi_{j,l+1}^{m+1} && {\scriptstyle (0\,\leqslant\, j\,\leqslant\, m+1,\ 0\,\leqslant\, l\,\leqslant\, m)}, \end{aligned}$$

and similarly, the matrix coefficients $\overline{\Pi}_{\bullet,\bullet}^{\bullet}$ *satisfy:*

$$
\begin{aligned}
-\beta\,\overline{\Pi}_{j-1,l}^{m} + \delta\,\overline{\Pi}_{j,l}^{m} &= \overline{\Pi}_{j,l}^{m+1} && \scriptstyle (0\leqslant j\leqslant m+1,\ 0\leqslant l\leqslant m),\\
\alpha\,\overline{\Pi}_{j-1,l}^{m} - \gamma\,\overline{\Pi}_{j,l}^{m} &= \overline{\Pi}_{j,l+1}^{m+1} && \scriptstyle (0\leqslant j\leqslant m+1,\ 0\leqslant l\leqslant m),
\end{aligned}
$$

Here by convention for all $j, l \in \mathbb{Z}$ and all $m \in \mathbb{N}$:

$$
\Big(j \leqslant -1 \quad \text{or} \quad m+1 \leqslant l \Big) \qquad\qquad \Longrightarrow \qquad\qquad \Pi_{j,l}^{m} := 0.
$$

Proof. To prove the first pair of identities, let us start from the fundamental identity:

$$
\sum_{0\leqslant j\leqslant m} R_j^m\,\overline{x}'^{\,m-j}\,\overline{y}'^{\,j} = \sum_{0\leqslant j\leqslant m} \overline{R}_j^m\,x'^{\,m-j}\,y'^{\,j},
$$

which we mutiply by $\overline{x}' = \alpha\,x' + \beta\,y'$:

$$
\begin{aligned}
\overline{x}' \sum_{0\leqslant j\leqslant m} R_j^m\,\overline{x}'^{\,m-j}\,\overline{y}'^{\,j} &= \big(\alpha\,x' + \beta\,y'\big) \sum_{0\leqslant j\leqslant m} \overline{R}_j^m\,x'^{\,m-j}\,y'^{\,j},\\
\sum_{0\leqslant j\leqslant m} R_j^m\,\overline{x}'^{\,m+1-j}\,\overline{y}'^{\,j} &= \sum_{0\leqslant j\leqslant m} \alpha\,\overline{R}_j^m\,x'^{\,m+1-j}\,y'^{\,j} + \sum_{0\leqslant j\leqslant m} \beta\,\overline{R}_j^m\,x'^{\,m-j}\,y'^{\,j}\\
&= \sum_{0\leqslant j\leqslant m+1} \Big(\alpha\,\overline{R}_j^m + \beta\,\overline{R}_{j-1}^m\Big)\,x'^{\,m+1-j}\,y'^{\,j},
\end{aligned}
$$

that is by definition of the matrices $\Pi_{\bullet,\bullet}^{m+1}$:

$$
\sum_{0\leqslant j\leqslant m+1} \bigg(\sum_{0\leqslant l\leqslant m} \Pi_{j,l}^{m+1}\,R_l^m \bigg)\,x'^{\,m+1-j}\,y'^{\,j} = \sum_{0\leqslant j\leqslant m+1} \bigg(\alpha \sum_{0\leqslant l\leqslant m} \Pi_{j,l}^{m}\,R_l^m + \beta \sum_{0\leqslant l\leqslant m} \Pi_{j-1,l}^{m}\,R_l^m \bigg)\,x'^{\,m+1-j}\,y'^{\,j}.
$$

By identification of the coefficients of $x'^{\,m+1-j}\,y'^{\,j}$, we therefore get:

$$
\sum_{0\leqslant l\leqslant m} \Pi_{j,l}^{m+1}\,R_l^m = \alpha \sum_{0\leqslant l\leqslant m} \Pi_{j,l}^{m}\,R_l^m + \beta \sum_{0\leqslant l\leqslant m} \Pi_{j-1,l}^{m}\,R_l^m \qquad \scriptstyle (0\leqslant j\leqslant m+1)
$$

and by identification of the coefficients of the R_l^m, we finally obtain the announced first collection of identities (the admitted convention being useful):

$$
\Pi_{j,l}^{m+1} = \alpha\,\Pi_{j,l}^{m} + \beta\,\Pi_{j-1,l}^{m} \qquad \scriptstyle (0\leqslant j\leqslant m+1).
$$

The second collection of identities is proved similarly, after multiplying by $\overline{y}' = \gamma\,x' + \delta\,y'$ instead of by $\overline{x}'$.

The second pair of collections of identities is proved totally similarly, by just dealing with the inverse transformation. □

Proof of the Miracle Factorizations 5.1. Applying these identities, we may confirm the first collection of 5 factorizations:

$$\begin{aligned}
\tau_{4-j,j}^{1,0} &= \overline{\Pi}_{j,0}^4\,\kappa + \overline{\Pi}_{j,1}^4\,\lambda + \overline{\Pi}_{j,2}^4\,\mu + \overline{\Pi}_{j,0}^4\,\nu \\
&= \Big(-\,\beta\,\overline{\Pi}_{j-1,0}^3 + \delta\,\overline{\Pi}_{j,0}^3\Big)\,\kappa + \Big(-\,\beta\,\overline{\Pi}_{j-1,1}^3 + \delta\,\overline{\Pi}_{j,1}^3\Big)\,\lambda + \Big(-\,\beta\,\overline{\Pi}_{j-1,2}^3 + \delta\,\overline{\Pi}_{j,2}^3\Big)\,\mu + \Big(-\,\beta\,\overline{\Pi}_{j-1,3}^3 + \delta\,\overline{\Pi}_{j,3}^3\Big)\,\nu \\
&= -\,\beta\,\Big[\overline{\Pi}_{j-1,0}^3\,\kappa + \overline{\Pi}_{j-1,1}^3\,\lambda + \overline{\Pi}_{j-1,2}^3\,\mu + \overline{\Pi}_{j-1,3}^3\,\nu\Big] \\
&\quad + \delta\,\Big[\overline{\Pi}_{j-1,0}^3\,\kappa + \overline{\Pi}_{j-1,1}^3\,\lambda + \overline{\Pi}_{j-1,2}^3\,\mu + \overline{\Pi}_{j-1,3}^3\,\nu\Big] \\
&= -\,\beta\,\big(\alpha\,\delta - \beta\,\gamma\big)^1\,(\cdots) + \delta\,\big(\alpha\,\delta - \beta\,\gamma\big)^1\,(\cdots) \\
&=: \big(\alpha\,\delta - \beta\,\gamma\big)^1\,\theta_{4-j,j}^{1,0},
\end{aligned}$$

while the second collection is handled similarly. □

Proposition 5.3. *In the affine chart $(x,y) \in \mathbb{C}^2 \subset \mathbb{P}^2$ minus the Jacobian zero-locus:*

$$\big\{D = 0\big\} \;=\; \big\{J = 0\big\} \cap \{T \neq 1\},$$

the coefficients $\big\{R_{i,j}\big\}_{i+j=4}$ and $S_{1,0}$, $S_{0,1}$, of any 2nd order logarithmic jet differential defined and holomorphic outside $\{abc = 0\}$ with total number of primes equal to $m = 4$:

$$\begin{aligned}
\mathsf{Jet}_{abc} \;:=\; &\sum_{i+j=4} R_{i,j}\,\Big(\big(\log\tfrac{a}{c}\big)'\Big)^i\,\Big(\big(\log\tfrac{b}{c}\big)'\Big)^j \\
&+ S_{1,0}\,\big(\log\tfrac{a}{c}\big)'\,\begin{vmatrix}\big(\log\frac{a}{c}\big)' & \big(\log\frac{b}{c}\big)' \\ \big(\log\frac{a}{c}\big)'' & \big(\log\frac{b}{c}\big)''\end{vmatrix} + S_{0,1}\,\big(\log\tfrac{b}{c}\big)'\,\begin{vmatrix}\big(\log\frac{a}{c}\big)' & \big(\log\frac{b}{c}\big)' \\ \big(\log\frac{a}{c}\big)'' & \big(\log\frac{b}{c}\big)''\end{vmatrix}
\end{aligned}$$

and with a, b, c allowed in denominator places, are rational expressions incorporating as maximal singularity:

$$\begin{aligned}
\frac{1}{D^4} &= \frac{1}{(\alpha\delta - \beta\gamma)^4} \qquad \textit{in}\;\; R_{i,j}, \\
\frac{1}{D^1} &= \frac{1}{(\alpha\delta - \beta\gamma)^2} \qquad \textit{in}\;\; S_{1,0},\;\; S_{0,1}..
\end{aligned}$$

□

6. Weighted Degree $m = 6$

For weighted degree $m = 5$, the maximal singularity is $\frac{1}{D^5}$, a property that relies upon factorizations of the shape:

$$\begin{aligned}
\tau_{5-j,j}^{2,0} &:= \overline{\Pi}_{j,0}^5\,\kappa + \overline{\Pi}_{j,1}^5\,\lambda + \overline{\Pi}_{j,2}^5\,\mu + \overline{\Pi}_{j,3}^5\,\nu = \big(\alpha\,\delta - \beta\,\gamma\big)^1\,\theta_{5-j,j}^{2,0} && {\scriptstyle (0 \leqslant j \leqslant 5)}, \\
\tau_{5-j,j}^{1,1} &:= \overline{\Pi}_{j,1}^5\,\kappa + \overline{\Pi}_{j,2}^5\,\lambda + \overline{\Pi}_{j,3}^5\,\mu + \overline{\Pi}_{j,4}^5\,\nu = \big(\alpha\,\delta - \beta\,\gamma\big)^1\,\theta_{5-j,j}^{1,2} && {\scriptstyle (0 \leqslant l \leqslant 5)}, \\
\tau_{5-j,j}^{0,2} &:= \overline{\Pi}_{j,2}^5\,\kappa + \overline{\Pi}_{j,3}^5\,\lambda + \overline{\Pi}_{j,4}^5\,\mu + \overline{\Pi}_{j,5}^5\,\nu = \big(\alpha\,\delta - \beta\,\gamma\big)^1\,\theta_{5-j,j}^{0,2} && {\scriptstyle (0 \leqslant l \leqslant 5)}.
\end{aligned}$$

At the next weighted degree $m = 6$, a slightly new phenomenon/difficulty occurs, while at all other weighted degrees, no more technical novelty/difficulty will be encountered.

Consider a jet differential of weighted order $m = 6$, written as:

$$
\begin{aligned}
\mathsf{Jet}_{abc} := & \sum_{i+j=6} R_{i,j} \Big(\big(\log \tfrac{a}{c}\big)'\Big)^i \Big(\big(\log \tfrac{b}{c}\big)'\Big)^j \\
& + \sum_{i+j=3} S_{i,j} \big(\log \tfrac{a}{c}\big)' \begin{vmatrix} \big(\log \frac{a}{c}\big)' & \big(\log \frac{b}{c}\big)' \\ \big(\log \frac{a}{c}\big)'' & \big(\log \frac{b}{c}\big)'' \end{vmatrix} \\
& + T_{0,0} \begin{vmatrix} \big(\log \frac{a}{c}\big)' & \big(\log \frac{b}{c}\big)' \\ \big(\log \frac{a}{c}\big)'' & \big(\log \frac{b}{c}\big)'' \end{vmatrix}^2,
\end{aligned}
$$

with a logarithmic Wronskian raised at power 2. To better organize calculations, we employ three different letters $R_{\bullet,\bullet}$, $S_{\bullet,\bullet}$, $T_{\bullet,\bullet}$.

Similarly as in the cases $m = 3, 4$, the transfer equation reads as:

$$
\begin{aligned}
& \sum_{i+j=6} \overline{R}_{i,j}\, x'^i\, y'^j + \sum_{i+j=3} \overline{S}_{i,j}\, W''' + \sum_{i+j=0} \overline{T}_{0,0}\, (W''')^2 \\
& = \sum_{i+j=6} R_{i,j} \, (\alpha\, x' + \beta\, y')^i \, (\gamma\, x' + \delta\, y')^j \\
& \quad + \sum_{i+j=3} S_{i,j} \, (\alpha\, x' + \beta\, y')^i \, (\gamma\, x' + \delta\, y')^j \, \Big(D\, W''' + \kappa\, x'^3 + \lambda\, x'^2 y' + \mu\, x' y'^2 + \nu\, y'^3\Big)^1 \\
& \quad + T_{0,0} \bigg(D^2\, (W''')^2 + 2\, D\, W''' \big[\kappa\, x'^3 + \lambda\, x'^2 y' + \mu\, x' y'^2 + \nu\, y'^3\big] + \big[\kappa\, x'^3 + \lambda\, x'^2 y' + \mu\, x' y'^2 + \nu\, y'^3\big]^2\bigg).
\end{aligned}
$$

An identification of the coefficients of $(W''')^2$ gives:

$$
\overline{T}_{0,0} = D^2\, T_{0,0}.
$$

An identification of the coefficients of W''' gives:

$$
\begin{aligned}
\sum_{i+j=3} \overline{S}_{i,j}\, x'^i\, y'^j = & \sum_{i+j=3} S_{i,j} \, (\alpha\, x' + \beta\, y')^i \, (\gamma\, x' + \delta\, y')^j \, D \\
& + 2\, D\, T_{0,0} \big[\kappa\, x'^3 + \lambda\, x'^2 y' + \mu\, x' y'^2 + \nu\, y'^3\big],
\end{aligned}
$$

that is, after reorganization and substitution of $T_{0,0}$:

$$
\sum_{i+j=3} S_{i,j} \, (\alpha\, x' + \beta\, y')^i \, (\gamma\, x' + \delta\, y')^j = \sum_{i+j=3} \frac{\overline{S}_{i,j}}{D}\, x'^i\, y'^j - 2\, \frac{\overline{T}_{0,0}}{D^2} \big[\kappa\, x'^3 + \lambda\, x'^2 y' + \mu\, x' y'^2 + \nu\, y'^3\big]
$$

Using the inverse matrix:

$$
\big(\Pi^3_{j,l}\big)^{-1} = \frac{1}{D^3}\, \big(\overline{\Pi}^3_{j,l}\big),
$$

and using again the 4 Miracle Factorizations 3.1 which compensates by a factor D^1 at numerator place the $\frac{1}{D^2}$ of $\frac{\overline{T}_{0,0}}{D^2}$, we realize that the maximal power of D present in all $S_{i,j}$ with $i + j = 3$ is equal not to 5 but to:

$$
4.
$$

Before making an identification of the coefficients of $\left(W'''\right)^0$, let us observe that by multiplying what precedes with $\kappa\,{x'}^3+\cdots+\nu\,{y'}^3$, we have the useful equation:

$$
\begin{aligned}
&\sum_{i+j=3} S_{i,j}\,\big(\alpha\,x'+\beta\,y'\big)^i\,\big(\gamma\,x'+\delta\,y'\big)^j\,\big[\kappa\,{x'}^3+\lambda\,{x'}^2y'+\mu\,x'{y'}^2+\nu\,{y'}^3\big]\\
&=\sum_{i+j=3}\frac{\overline{S}_{i,j}}{D}\,{x'}^i\,{y'}^j\,\big[\kappa\,{x'}^3+\lambda\,{x'}^2y'+\mu\,x'{y'}^2+\nu\,{y'}^3\big]-2\,\frac{\overline{T}_{0,0}}{D}\,\big[\kappa\,{x'}^3+\lambda\,{x'}^2y'+\mu\,x'{y'}^2+\nu\,{y'}^3\big]^2.
\end{aligned}
$$

Now, we can identify the terms which do not incorporate the Wronskian:

$$
\begin{aligned}
&\sum_{i+j=6} R_{i,j}\,\big(\alpha\,x'+\beta\,y'\big)^i\,\big(\gamma\,x'+\delta\,y'\big)^j\\
&=\sum_{i+j=6}\overline{R}_{i,j}\,{x'}^i\,{y'}^j-\sum_{i+j=3} S_{i,j}\,\big(\alpha\,x'+\beta\,y'\big)^i\,\big(\gamma\,x'+\delta\,y'\big)^j\,\big[\kappa\,{x'}^3+\lambda\,{x'}^2y'+\mu\,x'{y'}^2+\nu\,{y'}^3\big]\\
&\quad-\,T_{0,0}\,\big[\kappa\,{x'}^3+\lambda\,{x'}^2y'+\mu\,x'{y'}^2+\nu\,{y'}^3\big]^2,
\end{aligned}
$$

and hence, using the useful equation:

$$
\begin{aligned}
&\sum_{i+j=6} R_{i,j}\,\big(\alpha\,x'+\beta\,y'\big)^i\,\big(\gamma\,x'+\delta\,y'\big)^j\\
&=\sum_{i+j=6}\overline{R}_{i,j}\,{x'}^i\,{y'}^j-\sum_{i+j=3}\frac{\overline{S}_{i,j}}{D}\,{x'}^i\,{y'}^j\,\big[\kappa\,{x'}^3+\lambda\,{x'}^2y'+\mu\,x'{y'}^2+\nu\,{y'}^3\big]\\
&\qquad\qquad+(2-1)\,\frac{\overline{T}_{0,0}}{D^2}\,\big[\kappa\,{x'}^3+\lambda\,{x'}^2y'+\mu\,x'{y'}^2+\nu\,{y'}^3\big]^2.
\end{aligned}
$$

To determine the $\big\{R_{i,j}\big\}_{i+j=6}$ in terms of $\big\{\overline{R}_{i,j}\big\}_{i+j=6}$, $\big\{\overline{S}_{i,j}\big\}_{i+j=3}$, $\big\{\overline{T}_{i,j}\big\}_{i+j=0}$, we must now use the invere matrix:

$$
\big(\Pi_{j,l}^6\big)^{-1}\;=\;\frac{1}{D^6}\,\big(\overline{\Pi}_{j,l}^6\big),
$$

and, although we already know from an easy (not written) generalization to $m=6$ of the Miracle Factorizations 5.1 that the $\frac{1}{D}$ in the $\frac{\overline{S}_{i,j}}{D}$ will disappear, an new trouble occurs caused by the $\frac{1}{D^2}$ in $\frac{\overline{T}_{0,0}}{D^2}$. What to do, then? At least, expand the annoying square:

$$
\begin{aligned}
&\big[\kappa\,{x'}^3+\lambda\,{x'}^2y'+\mu\,x'{y'}^2+\nu\,{y'}^3\big]^2\\
&=\kappa^2\,{x'}^6+\big(2\,\kappa\,\lambda\big)\,{x'}^5y'+\big(2\,\kappa\,\mu+\lambda^2\big)\,{x'}^4{y'}^2+\big(2\,\kappa\,\nu+2\,\lambda\,\mu\big)\,{x'}^3{y'}^3+\big(2\,\lambda\,\nu+\mu^2\big)\,{x'}^2{y'}^4+\big(2\,\mu\,\nu\big)\,x'{y'}^5+\nu^2\,{y'}^6.
\end{aligned}
$$

At least, we suspect that there are more advanced

Miracle Factorizations 6.1. *One has:*

$$
\Big(\overline{\Pi}_{j,l}^6\Big)_{0\leqslant j\leqslant 6}^{0\leqslant l\leqslant 6}
\begin{pmatrix}
\kappa^2\\
2\,\kappa\,\lambda\\
2\,\kappa\,\mu+\lambda^2\\
2\,\kappa\,\nu+2\,\lambda\,\mu\\
2\,\lambda\,\nu+\mu^2\\
2\,\mu\,\nu\\
\nu^2
\end{pmatrix}
=
\begin{pmatrix}
D^2\,\big(\cdots\big)\\
D^2\,\big(\cdots\big)\\
D^2\,\big(\cdots\big)\\
D^2\,\big(\cdots\big)\\
D^2\,\big(\cdots\big)\\
D^2\,\big(\cdots\big)\\
D^2\,\big(\cdots\big)
\end{pmatrix}.
$$

Proof. Indeed, remembering the identity:

$$\kappa\,\overline{x}'^3 + \lambda\,\overline{x}'^2\overline{y}' + \mu\,\overline{x}'\overline{y}'^2 + \nu\,\overline{y}'^3 \;=\; \tau_{3,0}\,x'^3 + \tau_{2,1}\,x'^2y' + \tau_{1,2}\,x'y'^2 + \tau_{0,3}\,y'^3,$$

let us take the square:

$$\Big[\kappa\,\overline{x}'^3 + \lambda\,\overline{x}'^2\overline{y}' + \mu\,\overline{x}'\overline{y}'^2 + \nu\,\overline{y}'^3\Big]^2 \;=\; \Big[\tau_{3,0}\,x'^3 + \tau_{2,1}\,x'^2y' + \tau_{1,2}\,x'y'^2 + \tau_{0,3}\,y'^3\Big]^2,$$

that is:

$$\begin{aligned}
&\kappa^2\,\overline{x}'^6 + \big(2\,\kappa\,\lambda\big)\,\overline{x}'^5\overline{y}' + \big(2\,\kappa\,\mu + \lambda^2\big)\,\overline{x}'^4\overline{y}'^2 + \big(2\,\kappa\,\nu + 2\,\lambda\,\mu\big)\,\overline{x}'^3\overline{y}'^3 + \big(2\,\lambda\,\nu + \mu^2\big)\,\overline{x}'^2\overline{y}'^4 + \big(2\,\mu\,\nu\big)\,\overline{x}'\overline{y}'^5 + \nu^2\,\overline{y}'^6 \\
&\;=\; \tau_{3,0}^2\,x'^6 + \big(2\,\tau_{3,0}\,\tau_{2,1}\big)\,x'^5y' + \big(2\,\tau_{1,2}\,\tau_{3,0}\big)\,x'^4y'^2 + \big(2\,\tau_{0,3}\,\tau_{3,0} + 2\,\tau_{1,2}\,\tau_{2,1}\big)\,x'^3y'^3 \\
&\quad + \big(2\,\tau_{0,3}\,\tau_{2,1} + \tau_{1,2}^2\big)^2\,x'^2y'^4 + \big(2\,\tau_{1,2}\,\tau_{0,3}\big)\,x'y'^5 + \tau_{0,3}^2\,y'^6.
\end{aligned}$$

It only remains to replace on the left:

$$\begin{aligned}
\overline{x}' &\;=\; \alpha\,x' + \beta\,y', \\
\overline{y}' &\;=\; \gamma\,x' + \delta\,y',
\end{aligned}$$

and then by identification of the coefficients of these two sextic homogeneous polynomial in (x', y'), we obtain:

$$\Big(\overline{\Pi}_{j,l}^6\Big)_{0\leqslant j\leqslant 6}^{0\leqslant l\leqslant 6} \begin{pmatrix} \kappa^2 \\ 2\,\kappa\,\lambda \\ 2\,\kappa\,\mu + \lambda^2 \\ 2\,\kappa\,\nu + 2\,\lambda\,\mu \\ 2\,\lambda\,\nu + \mu^2 \\ 2\,\mu\,\nu \\ \nu^2 \end{pmatrix} \;=\; \begin{pmatrix} \tau_{3,0}^2 \\ 2\,\tau_{3,0}\,\tau_{2,1} \\ 2\,\tau_{1,2}\,\tau_{3,0} \\ 2\,\tau_{0,3}\,\tau_{3,0} + 2\,\tau_{1,2}\,\tau_{2,1} \\ 2\,\tau_{0,3}\,\tau_{2,1} + \tau_{1,2}^2 \\ 2\,\tau_{1,2}\,\tau_{0,3} \\ \tau_{0,3}^2 \end{pmatrix} \;=\; \begin{pmatrix} D^2\,(\cdots) \\ D^2\,(\cdots) \\ D^2\,(\cdots) \\ D^2\,(\cdots) \\ D^2\,(\cdots) \\ D^2\,(\cdots) \\ D^2\,(\cdots) \end{pmatrix},$$

thanks to:

$$\begin{pmatrix} \tau_{3,0} \\ \tau_{2,1} \\ \tau_{1,2} \\ \tau_{0,3} \end{pmatrix} \;=\; \begin{pmatrix} D\,(\cdots) \\ D\,(\cdots) \\ D\,(\cdots) \\ D\,(\cdots) \end{pmatrix}$$

which concludes. □

Thus, we realize that the maximal power of D present in all $R_{i,j}$ with $i + j = 6$ is equal to not to 8 but to:

$$6.$$

Proposition 6.2. *In the affine chart $(x, y) \in \mathbb{C}^2 \subset \mathbb{P}^2$ minus the Jacobian zero-locus:*

$$\big\{D = 0\big\} \;=\; \big\{J = 0\big\} \cap \{T \neq 1\},$$

the coefficients $\big\{R_{i,j}\big\}_{i+j=6}$, $\big\{S_{i,j}\big\}_{i+j=3}$, $\big\{T_{i,j}\big\}_{i+j=0}$, of any 2nd order logarithmic jet differential defined and holomorphic outside $\{abc = 0\}$ with total number of primes equal to

$m=4$:

$$
\begin{aligned}
\mathsf{Jet}_{abc} := & \sum_{i+j=6} R_{i,j} \left(\left(\log \tfrac{a}{c} \right)' \right)^i \left(\left(\log \tfrac{b}{c} \right)' \right)^j \\
& + \sum_{i+j=3} S_{i,j} \left(\log \tfrac{a}{c} \right)' \begin{vmatrix} \left(\log \tfrac{a}{c} \right)' & \left(\log \tfrac{b}{c} \right)' \\ \left(\log \tfrac{a}{c} \right)'' & \left(\log \tfrac{b}{c} \right)'' \end{vmatrix} \\
& + T_{0,0} \begin{vmatrix} \left(\log \tfrac{a}{c} \right)' & \left(\log \tfrac{b}{c} \right)' \\ \left(\log \tfrac{a}{c} \right)'' & \left(\log \tfrac{b}{c} \right)'' \end{vmatrix}^2,
\end{aligned}
$$

and with a, b, c allowed in denominator places, are rational expressions incorporating as maximal singularity:

$$
\begin{aligned}
\frac{1}{D^4} &= \frac{1}{(\alpha\delta - \beta\gamma)^6} \quad \textit{in } R_{i,j}, \\
\frac{1}{D^4} &= \frac{1}{(\alpha\delta - \beta\gamma)^4} \quad \textit{in } S_{i,j}, \\
\frac{1}{D^2} &= \frac{1}{(\alpha\delta - \beta\gamma)^2} \quad \textit{in } T_{0,0}.
\end{aligned}
$$

□

7. Weighted Degree $m \leqslant 13$, Jet Order 2

The general pattern of powers of D in denominator places is now easily devised. We will dispense us of writing formal, detailed proofs valuable for general $m \geqslant 1$ with jet order 2. According to the introduction of [1], we in fact only need to reach $m = 13$.

Thus, from an easily devised pattern and in the 3-conics case only (from now on) so that $2 = d_A = d_B = d_C$, a general logarithmic 2-jet differential of weighted order $m \leqslant 13$ must be of the following form in homogeneous coordinates:

$$
\begin{aligned}
\mathsf{Jet}_{ABC} = & \sum_{i+j=m} \frac{R_{i,j}(T,X,Y)}{\big(J(T,X,Y)\big)^m} \left((\log \frac{A}{C})' \right)^i \left((\log \frac{B}{C})' \right)^j \begin{vmatrix} \left(\log \tfrac{A}{C} \right)' & \left(\log \tfrac{B}{C} \right)' \\ \left(\log \tfrac{A}{C} \right)'' & \left(\log \tfrac{B}{C} \right)'' \end{vmatrix}^0 \\
& + \sum_{i+j=m-3} \frac{S_{i,j}(T,X,Y)}{\big(J(T,X,Y)\big)^{m-2}} \left((\log \frac{A}{C})' \right)^i \left((\log \frac{B}{C})' \right)^j \begin{vmatrix} \left(\log \tfrac{A}{C} \right)' & \left(\log \tfrac{B}{C} \right)' \\ \left(\log \tfrac{A}{C} \right)'' & \left(\log \tfrac{B}{C} \right)'' \end{vmatrix}^1 \\
& + \sum_{i+j=m-6} \frac{U_{i,j}(T,X,Y)}{\big(J(T,X,Y)\big)^{m-4}} \left((\log \frac{A}{C})' \right)^i \left((\log \frac{B}{C})' \right)^j \begin{vmatrix} \left(\log \tfrac{A}{C} \right)' & \left(\log \tfrac{B}{C} \right)' \\ \left(\log \tfrac{A}{C} \right)'' & \left(\log \tfrac{B}{C} \right)'' \end{vmatrix}^2 \\
& + \sum_{i+j=m-9} \frac{V_{i,j}(T,X,Y)}{\big(J(T,X,Y)\big)^{m-6}} \left((\log \frac{A}{C})' \right)^i \left((\log \frac{B}{C})' \right)^j \begin{vmatrix} \left(\log \tfrac{A}{C} \right)' & \left(\log \tfrac{B}{C} \right)' \\ \left(\log \tfrac{A}{C} \right)'' & \left(\log \tfrac{B}{C} \right)'' \end{vmatrix}^3 \\
& + \sum_{i+j=m-12} \frac{W_{i,j}(T,X,Y)}{\big(J(T,X,Y)\big)^{m-8}} \left((\log \frac{A}{C})' \right)^i \left((\log \frac{B}{C})' \right)^j \begin{vmatrix} \left(\log \tfrac{A}{C} \right)' & \left(\log \tfrac{B}{C} \right)' \\ \left(\log \tfrac{A}{C} \right)'' & \left(\log \tfrac{B}{C} \right)'' \end{vmatrix}^4,
\end{aligned}
$$

this formula being in fact valuable up to $m = 14$ (while for $m = 15$ one more line should be added), any sum $\sum_{i+j=\mathsf{negative}}$ being inexistent, with homogeneous polynomials:

$$
R_{\bullet,\bullet},\ S_{\bullet,\bullet},\ U_{\bullet,\bullet},\ V_{\bullet,\bullet},\ W_{\bullet,\bullet} \in \mathbb{C}[T,X,Y],
$$

of constant degrees:

$$d_R = \operatorname{deg} R_{\bullet,\bullet}, \quad d_S = \operatorname{deg} S_{\bullet,\bullet}, \quad d_U = \operatorname{deg} U_{\bullet,\bullet}, \quad d_V = \operatorname{deg} V_{\bullet,\bullet}, \quad d_W = \operatorname{deg} W_{\bullet,\bullet},$$

satisfying:

$$d_R - m \cdot 3 = d_S - (m-2) \cdot 3 = d_U - (m-4) \cdot 3 = d_V - (m-6) \cdot 3 = d_W - (m-8) \cdot 3 =: t,$$

so that this Jet_{ABC} is a *meromorphic* (rational) section of:

$$E_{2,m} T^*_{\mathbb{P}^2} \big(\log \mathscr{C} \big) \otimes \mathscr{O}_{\mathbb{P}^2}(t).$$

For Jet_{ABC} to really be a *holomorphic* logarithmic jet differential, the coefficients of the polynomials $R_{\bullet,\bullet}$, $S_{\bullet,\bullet}$, $U_{\bullet,\bullet}$, $V_{\bullet,\bullet}$, $W_{\bullet,\bullet}$, should satisfy a certain (huge) linear system which expresses that, when passing to any of the 3 affine charts $\{T \neq 0\}$ or $\{X \neq 0\}$ or $\{Y \neq 0\}$, and after simplifying all existing rational fractions, the (powers of the) Jacobian J in the denominator should disappear completely. More details about this appear in the Maple program copied below.

8. Key Vanishing Theorem

Key Vanishing Theorem 8.1. *For generic degree 2 (conic) homogeneous polynomials:*

$$A,\, B,\, C \in \mathbb{C}[T,X,Y]_2,$$

with the 3-conics divisor in $\mathbb{P}^2$:

$$\mathscr{C} = \big\{ A\, B\, C = 0 \big\},$$

there are no nonzero logarithmic 2-jet differentials:

$$0 = H^0 \Big(\mathbb{P}^2,\, E_{2,m} T^*_{\mathbb{P}^2} (\log \mathscr{C}) \otimes \mathscr{O}_{\mathbb{P}^2}(t) \Big),$$

for all (m,t) in the set:

$$\Big\{ (1,-1), (2,-1), (3,-3), (4,-3), (5,-4), (6,-5), (7,-5), (8,-6), (9,-7), (10,-7), (11,-8), (12,-9), (13,-9) \Big\}.$$

Proof. Let us take the equations of our 3 conics to be of the (very) simple Fermat-type form:

$$\begin{aligned} 0 &= A := 2\,T^2 + X^2 + Y^2, \\ 0 &= B := T^2 + 2\,X^2 + Y^2, \\ 0 &= C := T^2 + X^2 + 2\,Y^2, \end{aligned}$$

so that:

$$J = 32\, T X Y.$$

We will show that already for such (very) simple conics, there are no nonzero logarithmic 2-jet differentials for all the indicated values of (m,t). By semiconinuity, this will imply that the same holds true for a generic collection of 3 conics. The final computation will be delegated to a digital computer.

We will work in the 2 affine charts:

$$\big\{ T \neq 0 \big\} \qquad \text{and} \qquad \big\{ Y \neq 0 \big\},$$

replacing:

$$(T,X,Y) \longmapsto (1,x,y) \qquad \text{and} \qquad (T,X,Y) \longmapsto (t,x,1).$$

so that:

$$J = 32\,xy \qquad\qquad \text{and} \qquad\qquad J = 32\,tx.$$

In these two affine charts, the powers of $32\,xy$ and the powers of $32\,tx$ should completely disappear.

In the program copied below, we express the necessary divisibility by (powers of) xy and by (powers of) tx, we ask the computer to solve the concerned linear system. To speed up the process, we also work modulo a prime number p. It appears that with $p = 5$, the computations for all items (m,t) listed above always led us to conclude that the coefficients of all the polynomials $R_{\bullet,\bullet}$, $S_{\bullet,\bullet}$, $U_{\bullet,\bullet}$, $V_{\bullet,\bullet}$, $W_{\bullet,\bullet}$ are all zero.

The values of $1 \leqslant m \leqslant 14$ and of $t \in \mathbb{Z}$ can be changed manually on top of the program, to test (to treat) different items.

In the case $(m,t) = (13,-9)$, there were:

$$\mathbf{35\,378}\ \text{Equations},$$
$$\mathbf{12\,550}\ \text{Variables},$$

solved in:

$$\mathbf{22\,117}\ \text{Seconds},$$
$$\mathbf{994.27}\ \text{MegaOctets}.$$

□

References

[1] Hou, L.; Huynh, D.T.; Merker, J.; Xie, S.: *A second main theorem for entire curves intersecting three conics*, Preprint, 2026.

```
> restart: with(LinearAlgebra): with(Groebner):

  m := 13;

  twist := - m + 4;

  p := 5;

  dR := 3*(m-0) + twist:
  dS := 3*(m-2) + twist:
  dU := 3*(m-4) + twist:
  dV := 3*(m-6) + twist:
  dW := 3*(m-8) + twist:

  ##

  AA := 2*T^2 + 1*X^2 + 1*Y^2;
  BB := 1*T^2 + 2*X^2 + 1*Y^2;
  CC := 1*T^2 + 1*X^2 + 2*Y^2;

  #

  AT := diff(AA,T):  BT := diff(BB,T):  CT := diff(CC,T):
  AX := diff(AA,X):  BX := diff(BB,X):  CX := diff(CC,X):
  AY := diff(AA,Y):  BY := diff(BB,Y):  CY := diff(CC,Y):

  #

  ATT := diff(AA,T,T):  ATX := diff(AA,T,X):  ATY := diff(AA,T,Y):
  AXX := diff(AA,X,X):  AXY := diff(AA,X,Y):  AYY := diff(AA,Y,Y):
  BTT := diff(BB,T,T):  BTX := diff(BB,T,X):  BTY := diff(BB,T,Y):
  BXX := diff(BB,X,X):  BXY := diff(BB,X,Y):  BYY := diff(BB,Y,Y):
  CTT := diff(CC,T,T):  CTX := diff(CC,T,X):  CTY := diff(CC,T,Y):
  CXX := diff(CC,X,X):  CXY := diff(CC,X,Y):  CYY := diff(CC,Y,Y):

  #

  MatDD := Matrix([[AT,BT,CT],
                   [AX,BX,CX],
                   [AY,BY,CY]]):

  DD := factor(Determinant(MatDD));

  ##
```

```
Ap  := AT*T1 + AX*X1 + AY*Y1:
Bp  := BT*T1 + BX*X1 + BY*Y1:
Cp  := CT*T1 + CX*X1 + CY*Y1:

App  := + AT*T2 + AX*X2 + AY*Y2
      + ATT*T1*T1 + 2*ATX*T1*X1 + 2*ATY*T1*Y1 + AXX*X1*X1 + 2*
AXY*X1*Y1 + AYY*Y1*Y1:

Bpp  := + BT*T2 + BX*X2 + BY*Y2
      + BTT*T1*T1 + 2*BTX*T1*X1 + 2*BTY*T1*Y1 + BXX*X1*X1 + 2*
BXY*X1*Y1 + BYY*Y1*Y1:

Cpp  := + CT*T2 + CX*X2 + CY*Y2
      + CTT*T1*T1 + 2*CTX*T1*X1 + 2*CTY*T1*Y1 + CXX*X1*X1 + 2*
CXY*X1*Y1 + CYY*Y1*Y1:

##

logACp  := Ap/AA - Cp/CC:
logBCp  := Bp/BB - Cp/CC:

logACpp := App/AA - Ap^2/AA^2 - (Cpp/CC - Cp^2/CC^2):
logBCpp := Bpp/BB - Bp^2/BB^2 - (Cpp/CC - Cp^2/CC^2):

##

Carte_xy := {T =1,  X =x, Y =y,
           T1=0, X1=x1, Y1=y1,
           T2=0, X2=x2, Y2=y2}:

xyAA := subs(Carte_xy, AA):
xyBB := subs(Carte_xy, BB):
xyCC := subs(Carte_xy, CC):
xyDD := subs(Carte_xy, DD):

xylogACp := subs(Carte_xy, logACp):
xylogBCp := subs(Carte_xy, logBCp):

xylogACpp := subs(Carte_xy, logACpp):
xylogBCpp := subs(Carte_xy, logBCpp):

##

Carte_tx := {T =t,  X =x,  Y =1,
           T1=t1, X1=x1, Y1=0,
           T2=t2, X2=x2, Y2=0}:

txAA := subs(Carte_tx, AA):
```

```
txBB := subs(Carte_tx, BB):
txCC := subs(Carte_tx, CC):
txDD := subs(Carte_tx, DD):

txlogACp := subs(Carte_tx, logACp):
txlogBCp := subs(Carte_tx, logBCp):

txlogACpp := subs(Carte_tx, logACpp):
txlogBCpp := subs(Carte_tx, logBCpp):

##

Jet_ABC := +   (1/(T*X*Y)^(m-0))
                *
              add(R[m-0-j,j] * logACp^(m-0-j) * logBCp^j *
(logACp*logBCpp-logACpp*logBCp)^0, j=0..m-0)

          +    (1/(T*X*Y)^(m-2))
                *
              add(S[m-3-j,j] * logACp^(m-3-j) * logBCp^j *
(logACp*logBCpp-logACpp*logBCp)^1, j=0..m-3)

          +    (1/(T*X*Y)^(m-4))
                *
              add(U[m-6-j,j] * logACp^(m-6-j) * logBCp^j *
(logACp*logBCpp-logACpp*logBCp)^2, j=0..m-6)

          +    (1/(T*X*Y)^(m-6))
                *
              add(V[m-9-j,j] * logACp^(m-9-j) * logBCp^j *
(logACp*logBCpp-logACpp*logBCp)^3, j=0..m-9)

          +    (1/(T*X*Y)^(m-8))
                *
              add(W[m-12-j,j] * logACp^(m-12-j) * logBCp^j *
(logACp*logBCpp-logACpp*logBCp)^4, j=0..m-12):

##

xyJet_ABC := factor(+   (1/(x*y)^(m-0))
                  *
                add(Rxy[m-0-j,j] * xylogACp^(m-0-j) * xylogBCp^j
* (xylogACp*xylogBCpp-xylogACpp*xylogBCp)^0, j=0..m-0)

               +    (1/(x*y)^(m-2))
                  *
                add(Sxy[m-3-j,j] * xylogACp^(m-3-j) * xylogBCp^j
* (xylogACp*xylogBCpp-xylogACpp*xylogBCp)^1, j=0..m-3)
```

```
            +    (1/(x*y)^(m-4))
                 *
             add(Uxy[m-6-j,j] * xylogACp^(m-6-j) * xylogBCp^j
* (xylogACp*xylogBCpp-xylogACpp*xylogBCp)^2, j=0..m-6)

            +    (1/(x*y)^(m-6))
                 *
             add(Vxy[m-9-j,j] * xylogACp^(m-9-j) * xylogBCp^j
* (xylogACp*xylogBCpp-xylogACpp*xylogBCp)^3, j=0..m-9)

            +    (1/(x*y)^(m-8))
                 *
             add(Wxy[m-12-j,j] * xylogACp^(m-12-j) *
xylogBCp^j * (xylogACp*xylogBCpp-xylogACpp*xylogBCp)^4, j=0..
m-12)):

#

txJet_ABC := +   (1/(t*x)^(m-0))
                 *
             add(Rtx[m-0-j,j] * txlogACp^(m-0-j) * txlogBCp^j
* (txlogACp*txlogBCpp-txlogACpp*txlogBCp)^0, j=0..m-0)

            +    (1/(t*x)^(m-2))
                 *
             add(Stx[m-3-j,j] * txlogACp^(m-3-j) * txlogBCp^j
* (txlogACp*txlogBCpp-txlogACpp*txlogBCp)^1, j=0..m-3)

            +    (1/(t*x)^(m-4))
                 *
             add(Utx[m-6-j,j] * txlogACp^(m-6-j) * txlogBCp^j
* (txlogACp*txlogBCpp-txlogACpp*txlogBCp)^2, j=0..m-6)

            +    (1/(t*x)^(m-6))
                 *
             add(Vtx[m-9-j,j] * txlogACp^(m-9-j) * txlogBCp^j
* (txlogACp*txlogBCpp-txlogACpp*txlogBCp)^3, j=0..m-9)

            +    (1/(t*x)^(m-8))
                 *
             add(Wtx[m-12-j,j] * txlogACp^(m-12-j) *
txlogBCp^j * (txlogACp*txlogBCpp-txlogACpp*txlogBCp)^4, j=0..
m-12):

##

NFxy := proc(P,j,i)
```

```
expand(mtaylor(P,x,j) + mtaylor(P,y,i) - mtaylor(mtaylor(P,y,i),
x,j)):

end proc:

#

NFtx := proc(P,j,i)

expand(mtaylor(P,t,j) + mtaylor(P,x,i) - mtaylor(mtaylor(P,x,i),
t,j)):

end proc:
```

$$m := 13$$
$$twist := -9$$
$$p := 5$$
$$AA := 2\,T^2 + X^2 + Y^2$$
$$BB := T^2 + 2\,X^2 + Y^2$$
$$CC := T^2 + X^2 + 2\,Y^2$$
$$DD := 32\,T\,X\,Y \qquad (1)$$

```
> xyJet := mtaylor(

      factor((x*y*xyAA*xyBB*xyCC)^m * (subs(y2=(xyW+x2*y1)/x1,
xyJet_ABC)))

       , [x1,y1,xyW], 100):

#

txJet := mtaylor(

      factor((t*x*txAA*txBB*txCC)^m * (subs(x2=(txW+t2*x1)/t1,
txJet_ABC)))

       , [t1,x1,txW], 100):
```

```
> printlevel := 2:
  for k from 0 to m/3   do
  for j from 0 to m-3*k do

  xyEEEEq[m-j-3*k,j,k] := factor(coeftayl(xyJet, [x1,y1,xyW]=[0,0,
  0], [m-j-3*k,j,k])):

  od: od:

  #

  printlevel := 2:
  for k from 0 to m/3   do
  for j from 0 to m-3*k do

  txEEEEq[m-j-3*k,j,k] := factor(coeftayl(txJet, [t1,x1,txW]=[0,0,
  0], [m-j-3*k,j,k])):

  od: od:
```

```
> printlevel := 2:
  for k from 0 to m/3   do
  for j from 0 to m-3*k do

  xyEEEq[m-j-3*k,j,k] := mtaylor(

                        factor(
                             (1/(x^(m-j-3*k)*y^j*(x*y)^(3*k))) *
  (1/((xyAA*xyBB*xyCC)^(2*k)))
                               *
                             xyEEEEq[m-j-3*k,j,k])

  , [x,y], 1000):

  od: od:

  #

  printlevel := 2:
  for k from 0 to m/3   do
  for j from 0 to m-3*k do

  txEEEq[m-j-3*k,j,k] := mtaylor(
```

```
                        factor(
                              (1/(t^(m-j-3*k)*x^j*(t*x)^(3*k))) *
(1/((txAA*txBB*txCC)^(2*k)))
                                *
                              txEEEEq[m-j-3*k,j,k])

, [t,x], 1000):

od: od:
```

```
> printlevel := 2:
for k from 0 to m/3   do
for j from 0 to m-3*k do

xyEEq[m-j-3*k,j,k] := NFxy(xyEEEq[m-j-3*k,j,k], j,m-j-3*k) mod p:

od: od:

#

printlevel := 2:
for k from 0 to m/3   do
for j from 0 to m-3*k do

txEEq[m-j-3*k,j,k] := NFtx(txEEEq[m-j-3*k,j,k], j,m-j-3*k) mod p:

od: od:
```

```
> for j from 0 to m-0 do
Rxy[m-0-j,j] := add(add(cR[m-0-j,j,dR-q-r,q,r]*1^(dR-q-r)*x^q*
y^r, q=0..dR-r), r=0..dR):
od:

for j from 0 to m-3 do
Sxy[m-3-j,j] := add(add(cS[m-3-j,j,dS-q-r,q,r]*1^(dS-q-r)*x^q*
y^r, q=0..dS-r), r=0..dS):
od:

for j from 0 to m-6 do
Uxy[m-6-j,j] := add(add(cU[m-6-j,j,dU-q-r,q,r]*1^(dU-q-r)*x^q*
```

```
y^r, q=0..dU-r), r=0..dU):
od:

for j from 0 to m-9 do
Vxy[m-9-j,j] := add(add(cV[m-9-j,j,dV-q-r,q,r]*1^(dV-q-r)*x^q*
y^r, q=0..dV-r), r=0..dV):
od:

for j from 0 to m-12 do
Wxy[m-12-j,j] := add(add(cW[m-12-j,j,dW-q-r,q,r]*1^(dW-q-r)*x^q*
y^r, q=0..dW-r), r=0..dW):
od:

#

for j from 0 to m-0 do
Rtx[m-0-j,j] := add(add(cR[m-0-j,j,dR-q-r,q,r]*t^(dR-q-r)*x^q*
1^r, q=0..dR-r), r=0..dR):
od:

for j from 0 to m-3 do
Stx[m-3-j,j] := add(add(cS[m-3-j,j,dS-q-r,q,r]*t^(dS-q-r)*x^q*
1^r, q=0..dS-r), r=0..dS):
od:

for j from 0 to m-6 do
Utx[m-6-j,j] := add(add(cU[m-6-j,j,dU-q-r,q,r]*t^(dU-q-r)*x^q*
1^r, q=0..dU-r), r=0..dU):
od:

for j from 0 to m-9 do
Vtx[m-9-j,j] := add(add(cV[m-9-j,j,dV-q-r,q,r]*t^(dV-q-r)*x^q*
1^r, q=0..dV-r), r=0..dV):
od:

for j from 0 to m-12 do
Wtx[m-12-j,j] := add(add(cW[m-12-j,j,dW-q-r,q,r]*t^(dW-q-r)*x^q*
1^r, q=0..dW-r), r=0..dW):
od:
```

```
> printlevel := 2:
for k from 0 to m/3   do
for j from 0 to m-3*k do
```

```
xyEq[m-j-3*k,j,k] := NFxy(xyEEq[m-j-3*k,j,k], j,m-j-3*k):

od: od:

#

printlevel := 2:
for k from 0 to m/3   do
for j from 0 to m-3*k do

txEq[m-j-3*k,j,k] := NFtx(txEEq[m-j-3*k,j,k], j,m-j-3*k):

od: od:
```

```
> printlevel := 2:
for k from 0 to m/3   do
for j from 0 to m-3*k do

xyEquations[m-j-3*k,j,k] := {coeffs(xyEq[m-j-3*k,j,k], [x,y])}:

od: od:

#

printlevel := 2:
for k from 0 to m/3   do
for j from 0 to m-3*k do

txEquations[m-j-3*k,j,k] := {coeffs(txEq[m-j-3*k,j,k], [t,x])}:

od: od:
```

```
> xyEquations_Add := {}:

for k from 0 to m/3   do
for j from 0 to m-3*k do
```

```
xyEquations_Add := xyEquations_Add union xyEquations[m-j-3*k,j,k]
:

od: od:

Equations_xy := xyEquations_Add:

#

txEquations_Add := {}:

for k from 0 to m/3   do
for j from 0 to m-3*k do

txEquations_Add := txEquations_Add union txEquations[m-j-3*k,j,k]
:

od: od:

Equations_tx := txEquations_Add:
```

```
> Equations := Equations_xy union Equations_tx mod p:

  Nombre_Equations := nops(Equations);
```

$$Nombre\_Equations := 35378 \quad (2)$$

```
> Variables :=      {seq(seq(seq(cR[m-0-j,j,dR-q-r,q,r], q=0..dR-
  r), r=0..dR), j=0..m-0)}
            union {seq(seq(seq(cS[m-3-j,j,dS-q-r,q,r], q=0..dS-
  r), r=0..dS), j=0..m-3)}
            union {seq(seq(seq(cU[m-6-j,j,dU-q-r,q,r], q=0..dU-
  r), r=0..dU), j=0..m-6)}
            union {seq(seq(seq(cV[m-9-j,j,dV-q-r,q,r], q=0..dV-
  r), r=0..dV), j=0..m-9)}
           union {seq(seq(seq(cW[m-12-j,j,dW-q-r,q,r], q=0..dW-
  r), r=0..dW), j=0..m-12)}:

  Nombre_Variables := nops(Variables);
```

$$Nombre\_Variables := 12550 \qquad (3)$$

```
> Solutions := msolve(Equations,p):
```

```
> Verification := seq(op(2,op(i,Solutions)),i=1..nops(Solutions));
  indets({Verification}):
  nops(indets({Verification})):
```

$Verification :=$ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, **(4)**
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
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0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
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0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
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0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
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$0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0$

```
> assign(Solutions):
```

```
> for j from 0 to m-0 do
  Voir_R[m-0-j,j] := add(add(cR[m-0-j,j,dR-q-r,q,r]*T^(dR-q-r)*X^q*
  Y^r, q=0..dR-r), r=0..dR):
  od;

  for j from 0 to m-3 do
  Voir_S[m-3-j,j] := add(add(cS[m-3-j,j,dS-q-r,q,r]*T^(dS-q-r)*X^q*
  Y^r, q=0..dS-r), r=0..dS):
  od;

  for j from 0 to m-6 do
  Voir_U[m-6-j,j] := add(add(cU[m-6-j,j,dU-q-r,q,r]*T^(dU-q-r)*X^q*
  Y^r, q=0..dU-r), r=0..dU):
  od;

  for j from 0 to m-9 do
  Voir_V[m-9-j,j] := add(add(cV[m-9-j,j,dV-q-r,q,r]*T^(dV-q-r)*X^q*
  Y^r, q=0..dV-r), r=0..dV):
  od;

  for j from 0 to m-12 do
  Voir_W[m-12-j,j] := add(add(cW[m-12-j,j,dW-q-r,q,r]*T^(dW-q-r)*
  X^q*Y^r, q=0..dW-r), r=0..dW):
  od;
```

$$Voir\_R_{13,\,0} := 0$$
$$Voir\_R_{12,\,1} := 0$$
$$Voir\_R_{11,\,2} := 0$$
$$Voir\_R_{10,\,3} := 0$$
$$Voir\_R_{9,\,4} := 0$$
$$Voir\_R_{8,\,5} := 0$$
$$Voir\_R_{7,\,6} := 0$$
$$Voir\_R_{6,\,7} := 0$$

$$Voir\_R_{5,\,8} := 0$$
$$Voir\_R_{4,\,9} := 0$$
$$Voir\_R_{3,\,10} := 0$$
$$Voir\_R_{2,\,11} := 0$$
$$Voir\_R_{1,\,12} := 0$$
$$Voir\_R_{0,\,13} := 0$$
$$Voir\_S_{10,\,0} := 0$$
$$Voir\_S_{9,\,1} := 0$$
$$Voir\_S_{8,\,2} := 0$$
$$Voir\_S_{7,\,3} := 0$$
$$Voir\_S_{6,\,4} := 0$$
$$Voir\_S_{5,\,5} := 0$$
$$Voir\_S_{4,\,6} := 0$$
$$Voir\_S_{3,\,7} := 0$$
$$Voir\_S_{2,\,8} := 0$$
$$Voir\_S_{1,\,9} := 0$$
$$Voir\_S_{0,\,10} := 0$$
$$Voir\_U_{7,\,0} := 0$$
$$Voir\_U_{6,\,1} := 0$$
$$Voir\_U_{5,\,2} := 0$$
$$Voir\_U_{4,\,3} := 0$$
$$Voir\_U_{3,\,4} := 0$$
$$Voir\_U_{2,\,5} := 0$$
$$Voir\_U_{1,\,6} := 0$$
$$Voir\_U_{0,\,7} := 0$$
$$Voir\_V_{4,\,0} := 0$$
$$Voir\_V_{3,\,1} := 0$$
$$Voir\_V_{2,\,2} := 0$$
$$Voir\_V_{1,\,3} := 0$$
$$Voir\_V_{0,\,4} := 0$$
$$Voir\_W_{1,\,0} := 0$$
$$Voir\_W_{0,\,1} := 0 \tag{5}$$