Multilinear estimates for the Laplace spectral projectors on compact manifolds

Estimées multilinéaires pour les projecteurs spectraux du laplacien sur les variétés compactes

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Abstract

The purpose of this note is to extend to any space dimension the bilinear estimate for eigenfunctions of the Laplace operator on a compact manifold (without boundary) obtained in [1] in dimension 2. We also give some related trilinear estimates.

Résumé

L'objet de cette note est de généraliser à toute dimension d'espace les estimations bilinéaires de projecteurs spectraux de l'opérateur de Laplace sur une variété compacte (sans bord), démontrées dans [1] en dimension 2. On énonce aussi des estimations trilinéaires.

Version française abrégée

Soit (M, g) une variété riemanienne compacte, C^∞ (sans bord) et Δ le laplacien sur les fonctions de M. Nous avons obtenu précédemment ([1]) des estimées bilinéaires sur les projecteurs spectraux du laplacien

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dans le cas où la dimension de M est 2. Le but de cette note est de généraliser ces estimées en toute dimension d'espace :

Théorème 0.1 *Soit $\chi \in \mathcal{S}(\mathbb{R})$. Pour $\lambda \in \mathbb{R}$ on note $\chi_\lambda = \chi(\sqrt{-\Delta} - \lambda)$ le projecteur spectral autour de λ. Il existe C tel que pour tous $\lambda, \mu \geq 1$, $f, g \in L^2(M)$,*

$$\|\chi_\lambda f \, \chi_\mu g\|_{L^2(M)} \leq C\Lambda(d, \min(\lambda, \mu))\|f\|_{L^2(M)}\|g\|_{L^2(M)},$$

avec

$$\Lambda(d, \nu) = \begin{cases} \nu^{\frac{1}{4}} & \text{if } d = 2 \\ \nu^{\frac{d-2}{2}} \log^{1/2}(\nu) & \text{if } d = 3 \\ \nu^{\frac{d-2}{2}} & \text{if } d \geq 4 \end{cases}$$

De plus on a aussi pour tous $\lambda, \mu, \nu \geq 1$, $f, g, h \in L^2(M)$ l'estimation trilinéaire suivante :

$$\|\chi_\lambda f \, \chi_\mu g \, \chi_\nu h\|_{L^2(M)} \leq C \left(\frac{\lambda\mu\nu}{\max(\lambda, \mu, \nu)} \right)^{\frac{2d-3}{4}} \|f\|_{L^2(M)}\|g\|_{L^2(M)}\|h\|_{L^2(M)}.$$

Si on applique ce résultat en choisissant pour f et g deux harmoniques sphériques sur la sphère $\mathbb{S}^d$ on obtient

Corollaire 0.2 *Il existe $C > 0$ tel que si H_p et H_q sont deux harmoniques sphériques de degrés respectifs p et q plus grands que 1, on a*

$$\|H_p H_q\|_{L^2(\mathbb{S}^d)} \leq C\Lambda(d, \min(p, q))\|H_p\|_{L^2(\mathbb{S}^d)}\|H_q\|_{L^2(\mathbb{S}^d)} \tag{1}$$

La version *linéaire* de notre théorème ($\lambda = \mu = \nu$), sans la perte logarithmique pour $d = 3$, est due à Sogge [4,5,6]. En dimension 2, notre preuve dans [1] était inspirée par un travail de Hörmander [3] sur les opérateurs satisfaisant à la condition de Carleson-Sjölin. Ici notre preuve est différente (même pour $d = 2$) et repose sur une bilinéarisation des arguments de [4,5,6] dans l'esprit de Stein [7]. Plus précisément, après diverses réductions, on se ramène à établir deux estimées (micro)-locales *linéaires* sur l'opérateur. La première précise la continuité L^2 de l'opérateur, tandis que la seconde est fondée sur l'effet dispersif. Le fait que dans la relation (1) la plus haute fréquence disparaisse complètement du terme de droite est crucial en vue des applications à l'étude de l'équation de Schrödinger non linéaire (*cf.* [1]). Un calcul élémentaire sur les fonctions propres $e_n(x_1, x_2, x') = (x_1 + ix_2)^n$, $(x_1, x_2, x') \in \mathbb{S}^d$, qui se concentrent sur un équateur, montre que $p = 2$ est le plus grand indice pour lequel cette propriété est vraie si on étudie la norme L^p de $\chi_\lambda f \, \chi_\mu g$. Le même calcul montre que (1) est optimal sur $\mathbb{S}^2$ tandis que sur $\mathbb{S}^d$ ($d \geq 3$), l'optimalité (modulo la perte logarithmique) est obtenue en considérant les harmoniques sphériques *zonales* qui se concentrent sur deux pôles de la sphère.

D'autres applications du théorème 0.1 seront développées dans un article ultérieur.

1. Introduction

Let (M, g) be a compact smooth Riemannian manifold without boundary of dimension d and Δ be the Laplace operator on functions on M. In [1], we proved a bilinear estimate for the spectral projectors of Δ in the case $d = 2$. Our goal here is to extend this result to higher dimensions.

Theorem 1.1 *Let $\chi \in \mathcal{S}(\mathbb{R})$. For $\lambda \in \mathbb{R}$, denote by $\chi_\lambda = \chi(\sqrt{-\Delta} - \lambda)$ the spectral projector around λ. There exists C such that for any $\lambda, \mu \geq 1$, $f, g \in L^2(M)$,*

$$\|\chi_\lambda f \, \chi_\mu g\|_{L^2(M)} \leq C\Lambda(d, \min(\lambda, \mu))\|f\|_{L^2(M)}\|g\|_{L^2(M)}, \tag{2}$$

with

$$\Lambda(d,\nu)=\begin{cases}\nu^{\frac14} & \text{if } d=2\\ \nu^{\frac{d-2}{2}}\log^{1/2}(\nu) & \text{if } d=3\\ \nu^{\frac{d-2}{2}} & \text{if } d\geq 4\end{cases}$$

Moreover for any $\lambda,\mu,\nu\geq 1$, $f,g,h\in L^2(M)$, *the following trilinear estimate holds*

$$\|\chi_\lambda f\,\chi_\mu g\,\chi_\nu h\|_{L^2(M)}\leq C\left(\frac{\lambda\mu\nu}{\max(\lambda,\mu,\nu)}\right)^{\frac{2d-3}{4}}\|f\|_{L^2(M)}\|g\|_{L^2(M)}\|h\|_{L^2(M)}. \tag{3}$$

Applying this result with f and g two spherical harmonics on the sphere $\mathbb{S}^d$, we obtain

Corollary 1.2 *There exists* $C>0$ *such that if* H_p *and* H_q *are two spherical harmonics of respective degrees* p *and* q *greater than* 1,

$$\|H_pH_q\|_{L^2(\mathbb{S}^d)}\leq C\Lambda(d,\min(p,q))\|H_p\|_{L^2(\mathbb{S}^d)}\|H_q\|_{L^2(\mathbb{S}^d)} \tag{4}$$

The *linear* versions of our theorem ($\lambda=\mu=\nu$), without the logarithmic loss for $d=3$, are due to Sogge [4,5,6]. In the case $d=2$ our proof in [1] was inspired by Hörmander's work [3] on Carleson-Sjölin type operators. The proof we present here is different even for $d=2$ and relies on a bilinearization of the arguments in [4,5,6]. More precisely, after several reductions we reduce the matter to two (micro)-local *linear* estimates of quite a different nature. The fact that in (2) the highest frequency disappears completely in the right hand side of the estimate is crucial in the applications to the non-linear Schrödinger equation (see [1]). A simple computation on the eigenfunctions $e_n(x_1,x_2,x')=(x_1+ix_2)^n$, $(x_1,x_2,x')\in\mathbb{S}^d$, which concentrate on the equator, shows that $p=2$ is the highest index for which this phenomenon occurs if one studies the L^p norm of $\chi_\lambda f\,\chi_\mu g$. The same computation shows that (4) is optimal on $\mathbb{S}^2$. For $d\geq 3$, the optimality of (4) (except for the log loss) can be deduced by considering the *zonal* eigenfunctions which concentrate on two poles of the sphere.

We will give applications of Theorem 1.1 to the non linear Schrödinger equation in a forthcoming paper.

2. Bilinear Estimates

In this part we are going to give an outline of proof of the estimate (2). The proof of (3) is similar. We assume $1\leq\lambda\leq\mu$. The estimate (2) for any non trivial choice of χ implies (2) for all χ. Using a parametrix for the solution of the wave equations, Sogge [6] shows that for a suitable choice of the function χ, we have:

$$\chi_\lambda f=\lambda^{\frac{d-1}{2}}T_\lambda f+R_\lambda f,\qquad \|R_\lambda f\|_{L^\infty}\leq C\|f\|_{L^2}$$

and in a coordinate system close to $x_0\in M$,

$$T_\lambda f(x)=\int_{\mathbb{R}^d}e^{i\lambda\varphi(x,y)}a(x,y,\lambda)f(y)dy$$

where $a(x,y,\lambda)$ is a polynomial in λ^{-1} with smooth coefficients supported in the set $\{(x,y)\in\mathbb{R}^{2d};\,|x|\leq\delta\ll\varepsilon/C\leq|y|\leq C\varepsilon\}$ and $\varphi(x,y)=-d_g(x,y)$ where $d_g(x,y)$ is the geodesic distance between x and y.

In geodesic coordinates $y=\exp_0(r\omega)$, $r>0$, $\omega\in\mathbb{S}^{d-1}$, we have

$$T_\lambda f(x)=\int_0^\infty\int_{\omega\in\mathbb{S}^{d-1}}e^{i\lambda\varphi_r(x,\omega)}a_r(x,\omega,\lambda)f_r(\omega)drd\omega\overset{\text{def}}{=}\int_0^\infty T_\lambda^r f_r(x)dr.$$

where

$$dy=\kappa(r,\omega)drd\omega,\quad \varphi_r(x,\omega)=\varphi(x,r,\omega),\quad a_r(x,\omega,\lambda)=\kappa(r,\omega)a(x,r,\omega,\lambda),\quad f_r(\omega)=f(r,\omega).$$

We get

$$(T_\lambda f\, T_\mu g)(x) = \int_{\varepsilon/C}^{C\varepsilon}\int_{\varepsilon/C}^{C\varepsilon} T^r_\lambda f_r(x)\, T^q_\mu g_q(x) dr dq.$$

The operator T_λ is clearly bounded on $L^2(M)$ by $C\lambda^{-(d-1)/2}$ and the Minkowski inequality shows that to prove Theorem 1.1 it is enough to show, uniformly for $1 \leq \lambda \leq \mu$,

$$\|T^r_\lambda f\, T^q_\mu g\|_{L^2(M)} \leq C\Lambda(d,\lambda)(\lambda\mu)^{-\frac{(d-1)}{2}}\|f\|_{L^2(\mathbb{S}^{d-1})}\|g\|_{L^2(\mathbb{S}^{d-1})}. \tag{5}$$

We have

$$T^r_\lambda f\, T^q_\mu g(x) = \int_{\mathbb{S}^{d-1}\times\mathbb{S}^{d-1}} e^{i\lambda\varphi_r(x,\omega)+i\mu\varphi_q(x,\omega')} a_r(x,\omega,\lambda)a_q(x,\omega',\mu)f(\omega)g(\omega')d\omega d\omega'.$$

Here we see $\mathbb{S}^{d-1}$ as an embedded submanifold of $\mathbb{R}^d$ and consequently $\nabla_\omega\varphi_r(x,\omega) \in T_\omega\mathbb{S}^{d-1} \subset \mathbb{R}^d$. Therefore $\nabla_x\nabla_\omega\varphi_r$ is a function valued in $d\times d$ matrices. We work with ω' close to a point ω'_0 that we can assume to be equal to $(1,0,\dots,0)$. If M is a $d\times d$ matrix, we denote by $\Pi_1 M$ its first column and by $\Pi_1^\perp M$ its $d-1$ other columns.

Lemma 2.1 *Let $\omega'_0 = (1,0,\dots,0)$. For any $\alpha > 0$ there exist $c > 0$ such that if $|\omega_0 - \omega'_0| \geq \alpha$ and $|\omega_0 + \omega'_0| \geq \alpha$ then there exist $\varepsilon > 0$ such that if ω' is close to ω'_0, ω close to ω_0 and $\varepsilon/C \leq r,q \leq C\varepsilon$, then*

$$|\det\left[\Pi_1\nabla_x\nabla_\omega\varphi_r(x,\omega), \Pi_1^\perp\nabla_x\nabla_{\omega'}\varphi_q(x,\omega')\right]| \geq c > 0. \tag{6}$$

Proof. Proceeding as in [1] and using Gauss' lemma (see for example [2, 3.70]) we get

$$\nabla_x\varphi_r(0,\omega) = \omega. \tag{7}$$

Consequently, the matrix $\nabla_\omega\nabla_x\varphi_r(0,\omega)$ is the orthogonal projection onto the tangent plane to $\mathbb{S}^{d-1}$ at ω. Then, since we have supposed that $\omega_0 \neq \pm\omega'_0$,

$$\det\left[\Pi_1\nabla_x\nabla_\omega\varphi_r(0,\omega_0), \Pi_1^\perp\nabla_x\nabla_{\omega'}\varphi_q(0,\omega'_0)\right] \neq 0. \tag{8}$$

By continuity we get (6) for (x,ω,ω') close to $(0,\omega_0,\omega'_0)$ and $\varepsilon > 0$ small enough. □

Under the assumptions of Lemma 2.1, the function ω_1 is, in a neighbourhood of ω_0, a coordinate on $\mathbb{S}^{d-1}$ and we can consider now the operator (with frozen $\theta = (\omega_2,\dots,\omega_d)$)

$$T^{r,\theta}_\lambda f(x)\, T^q_\mu g(x) = \int_{\mathbb{R}\times\mathbb{S}^{d-1}} e^{i(\lambda\varphi_r(x,\omega_1,\theta)+\mu\varphi_q(x,\omega'))} a_r(x,\omega_1,\theta,\lambda)a_q(x,\omega',\mu)f(\omega_1)g(\omega')d\omega_1 d\omega'.$$

We can write

$$\|T^{r,\theta}_\lambda f\, T^q_\mu g\|^2_{L^2} = \int K(\omega_1,\sigma_1,\omega',\sigma')f(\omega_1)g(\omega')\overline{f(\sigma_1)g(\sigma)}d\omega_1 d\sigma_1 d\omega' d\sigma'$$

$$K(\omega_1,\sigma_1,\omega',\sigma') = \int_{x\in\mathbb{R}^d, |x|\leq\delta} e^{i\Psi_{r,q,\theta}(\omega_1,\sigma_1,\omega',\sigma',\lambda,\mu)} A_{r,q,\theta}(\omega_1,\sigma_1,\omega',\sigma',\lambda,\mu)dx$$

$$\Psi_{r,q,\theta}(x,\omega_1,\sigma_1,\omega',\sigma',\lambda,\mu) = \mu(\varphi_q(x,\omega') - \varphi_q(x,\sigma')) + \lambda(\varphi_r(x,\omega_1,\theta) - \varphi_r(x,\sigma_1,\theta))$$

where, according to Lemma 2.1 the phase Ψ satisfies

$$|\nabla_x\Psi| \geq C(\mu|\omega'-\sigma'| + \lambda|\omega_1-\sigma_1|), \qquad |\partial_x^\alpha\Psi| \leq C_\alpha(\mu|\omega'-\sigma'| + \lambda|\omega_1-\sigma_1|), \forall\alpha\in\mathbb{N}^d.$$

Using these estimates, by integrations by parts in x, we get easily

$$\forall N\in\mathbb{N}, \quad |K(\omega_1,\sigma_1,\omega',\sigma')| \leq C_N(1+\lambda|\omega_1-\sigma_1| + \mu|\omega'-\sigma'|)^{-N}.$$

Taking N large enough, Schur's Lemma gives

$$\|T_\lambda^{r,\theta} f\, T_\mu^q g\|_{L^2(M)}^2 \le C_N \left[\int_{t\in\mathbb{R},\rho\in\mathbb{R}^{d-1}} \frac{dt d\varrho}{(1+\lambda|t|+\mu|\varrho|)^N}\right] \|f\|_{L^2(\mathbb{S}^{d-1})}^2 \|g\|_{L^2(\mathbb{S}^{d-1})}^2$$
$$\le C\mu^{-(d-1)}\lambda^{-1}\|f\|_{L^2(\mathbb{S}^{d-1})}^2\|g\|_{L^2(\mathbb{S}^{d-1})}^2.$$

Using Minkowski inequality in the remaining $d-2$ coordinates on $\mathbb{S}_\omega^{d-1}$, and partitions of unity, this implies:

Proposition 2.1 *Suppose that $a_r(x,\omega,\lambda)a_q(x,\omega',\mu)$ is supported in a set where*

$$|\omega-\omega'|\ge\alpha \text{ and } |\omega+\omega'|\ge\alpha$$

and that q and r are close to each other. Then

$$\|T_\lambda^r f\, T_\mu^q g\|_{L^2(M)} \le C_\alpha \mu^{-\frac{d-1}{2}}\lambda^{-\frac{1}{2}}\|f\|_{L^2(\mathbb{S}^{d-1})}\|g\|_{L^2(\mathbb{S}^{d-1})}.$$

Remark that Proposition 2.1 gives (except for the log loss in dimension 3) the estimate (5) if $d\ge3$ and is better if $d=2$.

We are now left with the following two cases:

(i) a_r, a_q are localized close to $\omega=\omega_0$, $\omega'=\omega_0$ respectively.

(ii) a_r, a_q are localized close to $\omega=\omega_0$, $\omega'=-\omega_0$ respectively.

We are going to study the case (i), the case (ii) being similar. We follow Sogge's strategy [6], in the spirit of the arguments of Stein [7].

Lemma 2.2 *The phase $\varphi_q(x,\omega)$ is a Carleson-Sjölin phase: near any point (x_0,ω_0), one can choose a splitting of the variable $x=(t,z)$ such that*

(i) For fixed t the phase $\varphi_q(t,z,\omega)$ is uniformly non degenerate:

$$\left|\det\left(\frac{\partial^2\varphi_q(t,z,\omega)}{\partial z_j\partial\omega_i}\right)\right|\ge c>0. \tag{9}$$

(ii) Let $S_{t,z}=\{\nabla_{t,z}\varphi_q(t,z,\omega),\ \omega\sim\omega_0\}$. Then $S_{t,z}$ is according to (i) *a smooth hypersurface in $\mathbb{R}^d$ and it has non-vanishing principal curvatures: denote by $n(t,z,\omega)$ the normal unit vector to the surface S at the point $\nabla_{t,z}\varphi_q(t,z,\omega)$. Then*

$$\left|\det\langle\frac{\partial^2}{\partial\omega_j\partial\omega_i}\nabla_{t,z}\varphi_q(t,z,\omega),n(t,z,\omega)\rangle\right|\ge c>0. \tag{10}$$

Furthermore if r is close to q and ω_0 close to ω_0' or close to $-\omega_0'$, then we can choose the same splitting for the phases $\varphi_q(x,\omega)$ and $\varphi_r(x,\omega')$.

Proof. We may assume $\omega_0=(1,0,\dots,0)$. We choose $t=x_1$, $z=(x_2,\dots,x_d)$. Estimates (9) and (10) at the point $t=0,z=0$ are an easy consequence of (7) and the lemma follows by continuity. □

Using (9) we deduce the following refinement of the L^2 boundedness of the spectral projector.

Proposition 2.2 *The operator*

$$g\in L^2(M)\mapsto T_\nu^r g(t,z)\in L^\infty(\mathbb{R}_t;L^2(\mathbb{R}_z^{d-1}))$$

is continuous with norm bounded by $C\nu^{-(d-1)/2}$.

On the other hand, using the dispersion property (ii) in Lemma 2.2 leads to the following.

Proposition 2.3 *In the coordinate system of Lemma 2.2, we have:*

$$\|T_\nu^r f(t,z)\|_{L^2(\mathbb{R}_t;L^\infty(\mathbb{R}_z^{d-1}))}\le C\Lambda(d,\nu)\nu^{-(d-1)/2}\|f\|_{L^2}.$$

Before giving the proof of this result, let us show how to finish the proof of Theorem 1.1. Putting together Propositions 2.3 and 2.2 we get (using that the same splitting can be chosen for φ_r and φ_q) that

$$(f,g) \in L^2_\omega \times L^2_{\omega'} \mapsto T^r_\lambda f(t,z) \times T^q_\mu g(t,z) \in L^2(\mathbb{R}_t; L^2(\mathbb{R}^{d-1}_z))$$

is continuous with norm bounded by

$$C(\lambda\mu)^{-(d-1)/2}\Lambda(d,\lambda)$$

which is (5). □

Let us come to the proof of Proposition 2.3. By a TT^* argument, it is enough to estimate the norm of

$$T^r_\nu (T^r_\nu)^* : L^2(\mathbb{R}_{t'}; L^1(\mathbb{R}^{d-1}_{z'})) \mapsto L^2(\mathbb{R}_t; L^\infty(\mathbb{R}^{d-1}_z)).$$

But the kernel of this operator is

$$K(t,z,t',z') = \int e^{i\nu(\varphi_r(t,z,\omega)-\varphi_r(t',z',\omega))} a_r(t,z,\omega,\nu)\overline{a_r(t',z',\omega,\nu)} d\omega$$

and using the second part in Lemma 2.2 we can show that

$$|K(t,z,t',z')| \leq \frac{C}{(1+\nu|(t,z)-(t',z')|)^{(d-1)/2}} \leq \frac{C}{(1+\nu|t-t'|)^{(d-1)/2}}$$

Indeed,

$$\nabla_\omega \langle \nabla_{t,z}\varphi_r(t,z,\omega), n\rangle = 0 \Leftrightarrow n = n(t,z,\omega)$$

and taking into account that

$$\varphi_r(t,z,\omega) - \varphi_r(t',z',\omega) = \langle \nabla_{t,z}\varphi_r(t,z,\omega), (t,z)-(t',z')\rangle + \mathcal{O}_\omega(\|(t,z)-(t',z')\|^2),$$

we see that if $(t,z)-(t',z')$ is in a small conic neighbourhood of the critical direction $n(t,z)$ then, in view of (10), we can apply the stationnary phase formula to the integral in ω. Otherwise, we can integrate by parts in ω and using that

$$|\nabla_\omega (\varphi_r(t,z,\omega) - \varphi_r(t',z',\omega))| \geq c|(t,z)-(t',z')|$$

we get for any $N \in \mathbb{N}$ a better estimate (see [6, p 63]), namely,

$$|K(t,z,t',z')| \leq \frac{C_N}{(1+\nu|(t,z)-(t',z')|)^N}.$$

We conclude using the classical one dimensional Young inequality

$$\|T^r_\nu(T^r_\nu)^*\|_{\mathcal{L}\left(L^2(]-1,1[_{t'};L^1_{z'});L^2(]-1,1[_t;L^\infty_z)\right)} \leq C\int_{|s|\leq 2} \frac{ds}{(1+\nu|s|)^{(d-1)/2}} \leq \begin{cases} C\nu^{-1/2} & \text{if } d=2 \\ C\nu^{-1}\log(\nu) & \text{if } d=3 \\ C\nu^{-1} & \text{if } d\geq 4.\end{cases}$$

References

[1] N. Burq, P. Gérard and N. Tzvetkov Bilinear eigenfunction estimates and the non linear Schrödinger equation on surfaces *Preprint:* `http://www.arxiv.org/abs/math/0308214`, 2003.

[2] S. Gallot, D. Hulin J. Lafontaine, Riemannian geometry *Universitext, Springer-Verlag, Berlin*, 1990

[3] L. Hörmander. Oscillatory integrals and multipliers on FL^p. *Ark. Math.*, 11:1–11, 1973.

[4] C. Sogge. Oscillatory integrals and spherical harmonics. *Duke Math. Jour.*, 53:43–65, 1986.

[5] C. Sogge. Concerning the L^p norm of spectral clusters for second order elliptic operators on compact manifolds. *Jour. of Funct. Anal.*, 77:123–138, 1988.

[6] C. Sogge. Fourier integrals in classical analysis. *Cambridge tracts in Mathematics*, 1993.

[7] E.M. Stein Harmonic analysis: real-variable methods, orthogonality, and oscillatory integrals. *Monographs in Harmonic Analysis, III*, Princeton University Press, Princeton, NJ, 1993.