Séminaire : Problèmes spectraux en physique mathématique

Les séminaires ont lieu un lundi par mois, à l'**Institut Henri Poincaré**, 11 rue Pierre et Marie Curie, 75005 Paris.

Programme du 7 octobre 2013, en **salle 201** (2e étage)

— 11h15 - 12h15 : **Christophe Texier** (LPTMS, Orsay)
Products of random matrices of $\mathrm{SL}(2,\mathbb{R})$ **and 1D disordered systems**

Random matrix products are involved in many models of statistical physics, like random spin chains or quantum localisation problem. A central property of a random matrix product is the Lyapunov exponent characterising the growth of the product ; the physical interpretation is the free energy per spin of a spin chain, or the inverse localisation length in quantum localisation problems.
I will consider products of i.i.d. random real 2×2 matrices and will identify a physical model of one-dimensional disordered quantum mechanics related to the most general matrix products. Using the Iwasawa decomposition of $\mathrm{SL}(2,\mathbb{R})$, we can identify a continuum regime where the mean values and the covariances of the three Iwasawa parameters are simultaneously small (matrices are close to the identity matrix). In this regime, the Lyapunov exponent of the product may be systematically obtained by considering the Hilbert transform of the invariant measure of the matrix product. The Lyapunov exponent is shown to present a scaling form.
This general analysis allows us, among other things, to recover in a unified framework few results known previously from exactly solvable models of one-dimensional disordered systems and find several new ones, providing a classification of possible solutions for such 1D models.
References :
* A.Comtet, C.Texier & Y.Tourigny, *Products of random matrices and generalised quantum point scatterers*, J. Stat. Phys. **140**(3), 427–466 (2010).
* A.Comtet, J.-M.Luck, C.Texier & Y.Tourigny, *The Lyapunov exponent of products of random* 2×2 *matrices close to the identity*, J. Stat. Phys. **150**(1), 13–65 (2013).

— 14h - 15h : **Tiphaine Jézéquel** (ENS Cachan Bretagne)
Klein-Gordon non linéaire : dynamique près d'une orbite homocline

On s'intéresse à l'équation de Klein-Gordon non-linéaire :

$$\partial_t^2 u - \Delta u - m^2 u + u^{2p+1} = 0, \quad (t,x) \in \mathbb{R}\times M,$$

où M est une variété Riemanienne compacte de dimension 1, 2 ou 3. Si l'on s'intéresse d'abord au comportement de l'équation stationnaire en x, le portrait de phase est entièrement traçable, et notamment cette équation admet les équilibres 0 et $m^{\frac{1}{p}}$, ainsi que 2 solutions ondes solitaires à 0 (appelées aussi orbites homoclines à 0).
Notre principal résultat est l'existence d'un grand nombre de solutions non stationnaires en x proches de ces ondes solitaires. On souligne qu'on décrit ainsi des comportements à la fois non-linéaires et non locaux pour cette équation. Bien que non locale, la démonstration est inspirée d'une stratégie utilisée par Groves et Schneider dans l'étude d'une bifurcation locale de Klein-Gordon.
Il s'agit d'un travail en collaboration avec Benoît Grébert et Laurent Thomann.

— 15h15 - 16h15 : **Thierry Daudé** (Cergy-Pontoise)
Inverse scattering at fixed energy in black hole spacetimes

In this talk, we first describe a class of axisymmetric, electrically charged, spacetimes with positive cosmological constant, called Kerr-Newmann-de-Sitter black holes, which are exact solutions of the Einstein equations. The main question we address is the following : can we determine the metrics of such black holes by observing waves at the "infinities" of the spacetime ? Precisely, the considered waves will be massless Dirac fields evolving in the outer region of Kerr-Newman-de-Sitter black holes. We shall define the corresponding scattering matrix, the object that encodes the far field behavior of these Dirac fields from the point of view of static observers. We finally shall show that the metrics of such black holes is uniquely determined by the knowledge of this scattering matrix at a fixed energy.
This result was obtained in collaboration with François Nicoleau (Nantes).

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