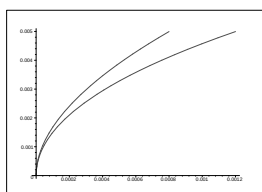


Feuille d'exercices de mathématiques n°3

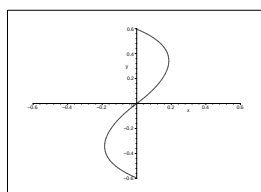
1. Etudier au voisinage de 0 les courbes suivantes.

- a) $t \mapsto \begin{pmatrix} 1+t^2 \\ t^2(1-t) \end{pmatrix}$. b) $t \mapsto \begin{pmatrix} 1+t^2 \\ t^2(1-t^5) \end{pmatrix}$.
c) $t \mapsto \begin{pmatrix} t \cos(t) \\ t \sin(t) \end{pmatrix}$. d) $t \mapsto \begin{pmatrix} t^2 \cos(t) \\ t \sin(t) \end{pmatrix}$.
e) $t \mapsto \begin{pmatrix} t^2 \cos(t) \\ t^2 \sin(t) \end{pmatrix}$. f) $t \mapsto \begin{pmatrix} (1+t)\ln(1-t) + t \\ -1 + \cos(t) \end{pmatrix}$.

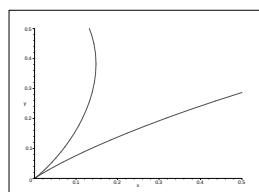
2. Soient $\gamma_1, \gamma_2, \gamma_3, \gamma_4$ et γ_5 les courbes paramétrées définies par $\gamma_1(t) = \begin{pmatrix} t^3 - t^5 \\ \sin^3 t \end{pmatrix}$, $\gamma_2(t) = \begin{pmatrix} 10t^4 + 20t^5 \\ 1 - \cos(t) \end{pmatrix}$, $\gamma_3(t) = \begin{pmatrix} -\sin^3(t) \\ \sin^2(t) \cos(t) \end{pmatrix}$, $\gamma_4(t) = \begin{pmatrix} t^2 + t^3 \\ \sin^2(t) \end{pmatrix}$, $\gamma_5 = \begin{pmatrix} 1 - \cos(t) \\ t - \sin(t) \end{pmatrix}$. En raisonnant à l'aide de développements limités, indiquer quelles sont les courbes partiellement représentées par les graphiques ci-dessous.



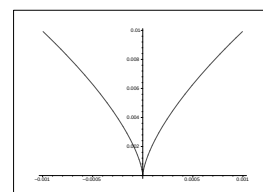
(a)



(b)



(c)

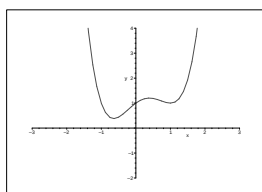


(d)

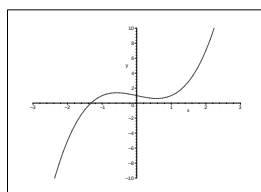
3. On a représenté ci-dessous les graphes des 10 polynômes suivants.

$$P_1(x) = (x+1)(x-1) + 2, \quad P_2(x) = (x-1)x(x+1) + 1, \quad P_3(x) = (x+1)^3 - 2, \quad P_4(x) = x^4, \\ P_5(x) = (x^2+1)(x-2)(x+2) - 2, \quad P_6(x) = (x-2)(x-1)x(x+2) + 1, \quad P_7(x) = (x-1)^2x(x+1) + 1, \\ P_8(x) = x^5 - 2x^2 + 5x + 1, \quad P_9(x) = x^5 + x^3 - 6x, \quad P_{10}(x) = (x-2)(x-1)x(x+1)(x+2).$$

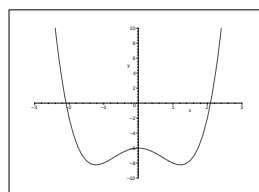
Identifier par des arguments mathématiques les plus simples possibles à quel polynôme correspond chaque graphique.



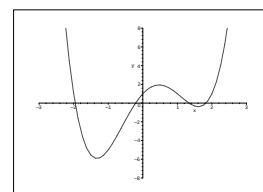
(a)



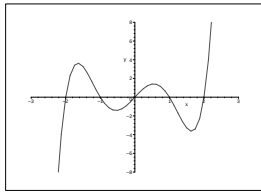
(b)



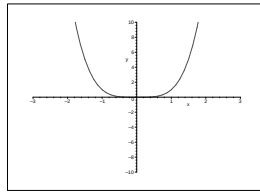
(c)



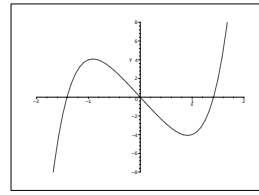
(d)



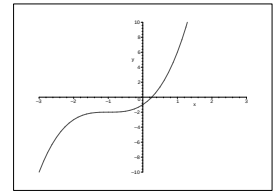
(e)



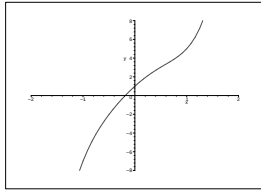
(f)



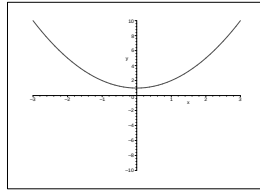
(g)



(h)



(i)



(j)

4. Etudier et construire la courbe paramétrée définie sur $\mathbf{R}$ par $t \mapsto \left(\frac{1}{2}t^2 - t, \frac{1}{3}t^3 - \frac{1}{2}t^2 \right)$.
5. Etudier et construire la courbe paramétrée définie sur $\mathbf{R} \setminus \{0, 1\}$ par $t \mapsto \left(\frac{t^2-1}{t+1}, \frac{t}{t(t-1)} \right)$.