

Test 1

Ex 2. $X \sim U_b(0, 2)$, $f(x) = \begin{cases} \frac{1}{2} & \text{if } x \in \{0, 2\} \\ 0 & \text{otherwise} \end{cases}$

$$\text{C) } F(x) = \int_{-\infty}^x f(u) du = \begin{cases} \int_0^x \frac{1}{2} du = \frac{x}{2} & \text{if } x \in \{0, 2\} \\ 0 & \text{if } x \leq 0 \\ 1 & \text{if } x \geq 2 \end{cases}$$

The median^m is the 0.5-quantile of the distribution of X :

$$\begin{aligned} m &= (CDF)^{-1}(0.5) \\ &= 2 \times 0.5 \quad \left(CDF^{-1}(y) = 2y \right) \\ &= 1 \quad \left(y \in \{0, 1\} \right) \end{aligned}$$

$q = 0.1$ -quantile: $q = 2 \times 0.1 = 0.2$

Ex 3 Method of moments:

1/ Write θ as a function of $E(X)$:

$$EX = \frac{\theta}{\theta - 1} \Leftrightarrow (EX)(\theta - 1) = \theta \Leftrightarrow \theta = \frac{EX}{EX - 1}$$

2/ Replace EX by the sample mean:

$$\hat{\theta}_{MM} = \frac{\bar{x}}{\bar{x} - 1}$$