SUMMARY

Let (M, ω) be a symplectic manifold compact or convex at infinity. Consider a closed Lagrangian submanifold L such that

$$\omega|_{\pi_2(M,L)} = 0 \text{ and } \mu|_{\pi_2(M,L)} = 0$$

where μ is the Maslov index. Given any Lagrangian submanifold L', transverse and Hamiltonian isotopic to L, we define Lagrangian spectral numbers of order 2 associated to each non zero homology class of L, using techniques similar to the ones used by Schwarz in the Hamiltonian case. We show that these numbers only depend on the two Lagrangian submanifolds L and L' (Theorem 4.5) and that they naturally extend the Hamiltonian spectral invariants introduced by Oh and Schwarz (Proposition 4.7).

Moreover, we introduce higher order spectral numbers via spectral sequence machinery introduced by Barraud and Cornea. These invariants are new even in the Hamiltonian case and, as an example, we compute them explicitely in the Morse case, for $L = (S^2 \times S^4)\#(S^2 \times S^4)$, showing that they carry non trivial information even in the Morse case. This example leads obviously to a symplectic one, when extended to the cotangent bundle of L.

We show that the order 2 spectral invariants are their homological counterparts. We provide a way to distinguish our higher order Lagrangian spectral invariants one from the other via a purely topological object and estimate their difference in terms of a geometric quantity only depending on the geometry of the two fixed Lagrangian submanifolds L and L' (Theorem 4.24). This is the main

result of our work and leads us to interesting consequences with respect to the order 2 spectral invariants. Moreover we get a bound for the Hofer's distance between L and L' in terms of the cup-length of L and our geometric quantity which is shown to improve the classical bound.

Keywords – Symplectic topology and geomety, Lagrangian submanifolds and Lagrangian intersections, spectral sequences, Hofer's distance, action functional, Morse and Floer homologies, spectral invariants.