

# Séminaire : Problèmes spectraux en physique mathématique

Les séminaires ont lieu à l'**Institut Henri Poincaré**, 11 rue Pierre et Marie Curie, Paris.

Programme du lundi 12 mars 2018, en **amphi Hermite** (RdC)

— 11h15 - 12h15 : **Mireille Capitaine** (Toulouse)

**Outliers de modèles matriciels Hermitiens polynomiaux.**

Le but de cet exposé est de dégager une méthodologie générale pour localiser, en grande dimension, les éventuelles valeurs propres s'éloignant du reste du spectre ("outliers") de polynômes auto-adjoints non commutatifs en des matrices aléatoires asymptotiquement libres.

— 14h - 15h : **Peng Zhou** (IHES)

**Zero set of eigenfunction for harmonic oscillators.**

Eigenfunctions of the Laplacian on a smooth Riemannian manifold are a classical object of study. One of Yau's conjecture predicts that the 'size' of the nodal set of the  $N$ -th eigenfunction grows as  $N^{1/2}$ . In this talk, we present the analogous study for the Schrödinger equation, in the simplest example of the harmonic oscillator on  $\mathbb{R}^n$ . For a fixed total energy  $E$ , the configuration space is partitioned into an 'allowed region' and a 'forbidden region', and roughly speaking, the eigenfunction oscillates in the allowed region and exponentially decays in the forbidden region. In the semiclassical limit, the nodal sets exhibit different behaviors in the two regions and across the interface.

This is joint work with Steve Zelditch and Boris Hanin. (arXiv 1310.4532, arXiv 1602.06848)

— 15h15 - 16h15 : **Vladimir Lotoreichik** (Prague)

**A Faber-Krahn inequality for the Robin Laplacian on exterior domains.**

We will discuss several generalizations of the Faber-Krahn inequality for the lowest eigenvalue of the Robin Laplacian with a negative boundary parameter on the exterior of a bounded, simply connected, smooth domain  $\Omega \subset \mathbb{R}^d$ . Some further generalizations for disconnected  $\Omega$ 's will also be discussed. Our main motivation is to go beyond more traditional bounded domains in eigenvalue optimization.

The ultimate goal is always to prove that the exterior of a ball maximizes the underlying lowest eigenvalue under a suitable constraint being imposed. In two dimensions, we constrain either the perimeter of  $\Omega$  or its area. In higher dimension, constraining the area of  $\partial\Omega$  leads to an ill-posed optimization problem, as the large coupling asymptotics in PANKRASHKIN-POPOFF-16 shows. Instead, we constrain for  $d \geq 3$  the ratio between a Willmore-type energy of  $\partial\Omega$  and the area of  $\partial\Omega$ , assuming, additionally, that  $\Omega$  is convex.

In the proofs we represent the lowest eigenvalue via the min-max principle on the level of quadratic forms expressed in suitably chosen coordinates on  $\mathbb{R}^d \setminus \overline{\Omega}$ . We make use either of standard parallel coordinates or of their modification worked out in PAYNE-WEINBERGER-61 and further refined in SAVO-01. The trickiest part of the proof is to find a proper test function.

These results are obtained in collaboration with David Krejčířík.

Pour tout renseignement, contacter les organisateurs

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