Séminaire : Problèmes spectraux en physique mathématique

Les séminaires ont lieu à l'**Institut Henri Poincaré**, 11 rue Pierre et Marie Curie, 75005 Paris.

Programme du lundi 19 octobre 2015, en **salle 314**

— 11h15 - 12h15 : **Mariapia Palombaro** (Univ. of Sussex)
Homogenization of the Schrödinger equation in a locally periodic medium
I will present a localization result for the Schrödinger equation in a locally periodic medium. For the time and space scaling of semi-classical analysis we consider well-prepared initial data that are concentrated near a stationary point of the energy. By the method of two-scale convergence, we show that there exists a localized solution which is asymptotically given as the product of a Bloch wave and of the solution of an homogenized Schrödinger equation with quadratic potential. In the last part of the talk I will also discuss some results, obtained by the same method, on the diffraction of Bloch wave packets in periodic media over long times.
The results have been obtained in joint work with Grégoire Allaire and Jeffrey Rauch.

— 14h - 15h : **Georgi Vodev** (Nantes)
Asymptotic behavior of the interior transmission eigenvalues
Let $\Omega \subset \mathbf{R}^d$ be a bounded connected domain with a smooth boundary $\Gamma = \partial\Omega$. A nonvanishing number $\lambda \in \mathbf{C}$ will be said to be a transmission eigenvalue if the following problem has a non-trivial solution :

$$\begin{cases} (\nabla c_1(x)\nabla + \lambda n_1(x))\, u_1 = 0 & \text{in} \quad \Omega, \\ (\nabla c_2(x)\nabla + \lambda n_2(x))\, u_2 = 0 & \text{in} \quad \Omega, \\ u_1 = u_2, \;\; c_1\partial_\nu u_1 = c_2\partial_\nu u_2 & \text{on} \quad \Gamma, \end{cases}$$

where ν denotes the exterior Euclidean unit normal to Γ, $c_j, n_j \in C^\infty(\overline{\Omega})$ are strictly positive real-valued functions. We will discuss two questions :
1. The localization of the transmission eigenvalues on the complex plane ;
2. The asymptotic behaviour of the counting function $N(r) = \#\{\lambda - \text{trans. eig. : } |\lambda| \leq r^2\}$, as $r \to \infty$.

Among other things, we prove that $N(r)$ satisfies the asymptotics $N(r) = (\tau_1 + \tau_2)r^d + O_\varepsilon(r^{d-\kappa+\varepsilon})$, where $0 < \kappa \leq 1$ is such that there are no transmission eigenvalues in the region $\{|\mathrm{Im}\,\lambda| \geq C\,(|\mathrm{Re}\,\lambda| + 1)^{1-\frac{\kappa}{2}}\}$, and $\tau_j = \frac{\omega_d}{(2\pi)^d}\int_\Omega \left(\frac{n_j(x)}{c_j(x)}\right)^{d/2} dx$, with ω_d the volume of the d-dimensional unit ball.

— 15h15 - 16h15 : **Yannick Privat** (UPMC)
Optimal shape and location of actuators or sensors in PDE models
We investigate the problem of optimizing the shape and location of actuators or sensors for evolution systems driven by a partial differential equation, like for instance a wave equation, a Schrödinger equation, or a parabolic system, on an arbitrary domain Ω, in arbitrary dimension, with boundary conditions if there is a boundary, which can be of Dirichlet, Neumann, mixed or Robin. This kind of problem is frequently encountered in applications where one aims, for instance, at maximizing the quality of reconstruction of the solution, using only a partial observation. From the mathematical point of view, using probabilistic considerations we model this problem as the problem of maximizing what we call a randomized observability constant, over all possible subdomains of Ω having a prescribed measure. The spectral analysis of this problem reveals intimate connections with the theory of quantum chaos. More precisely, if the domain Ω satisfies some quantum ergodic assumptions then we provide a solution to this problem.
This work is in collaboration with Emmanuel Trélat (Univ. Paris 6) and Enrique Zuazua (BCAM